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Research Article

On Order Selection of Arima Models for Stationary Normal and Non-Normal Data Structures

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ABSTRACT

The study is aimed at identifying the orders of time series models in stationary and non-stationary non-normal data from different distributions; with a view to determining the best Autoregressive/ Moving Average orders from different time series, ARIMA models were considered for different underline distributions of data. The data is generated under normal uniform and exponential distribution using a second-order autoregressive model. Data were generated in two forms, these are, when stationarity is observed and when it is violated. Each case of data simulated is fitted to different models and the values of AIC, BIC, HQIC, and FPE are computed. The effect of different levels of parameters (0.3, 0.6, and -0.3, -0.6) at the sample size of 20, 40, 60, 80, 100, 120, 140, 160, 180, and 200 which we considered to represent moderate and large sample sizes respectively on the simulated data from the stationary normal and non-normal data. We concluded in general that the selections of the order for the models considered in this study are tied more to the underlying distribution of the series in relation to the Stationarity and non-Stationarity of the series as it will lead to the identification of the proper model. Since the selection are almost identical in the stationary and non-stationary from both normal and non-normal data structure but varies with the variation in the distribution of the series. And it was also observed that for the ARIMA models the order stocked to the principle of parsimony i.e. models with lower-order selected at most of the sample sizes considered. The need to develop a methodology for model selection that combines both objective and subjective techniques is strongly recommended.

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INTRODUCTION

Model identification in time series modeling and forecasting dates back to the pioneering of Box and Jenkins in 1976; Identifying an appropriate parameter of the tentative model is very paramount if the true objective of the modeling is to be achieved. Box-Jenkins's method is a tripartite step of the Identification, Estimation, and Diagnostic/Forecasting approach in time series modeling. The idea behind Box-Jenkins's approach is to generalize the true data generating process based on the observed values (Stadnytska *et al.,* 2008). Of the three iterative procedures of Box-Jenkins's approach, identification gives an overall insight into the basic properties (parameters) of the tentative model of the series under investigation. Identifying the proper model of the time series data is extremely significant if a better understanding of the whole process is to be achieved (kirushanthini, 2019). Wu and Drignei, (2021) proposed an order identification algorithm for big-time series data. The algorithm proposed is for integer order, using the kriging-based method to emulate the information criteria such as BIC on the rest of the grid and they utilized Efficient Global Optimization (EGO) to identify the orders. Their proposed algorithm is applicable to both ARMA and ARMA-GARCH

Box-Jenkins has received tremendous and wide application in various fields of science, Behavioral Sciences, Engineering, and many different areas (see, Goin and Ahern, 2019; Green, 2011; Fokianos and Kedem, 2002; Chen, Min, and Chen, 2013) Several techniques for identifying the tentative model have been discussed extensively in the literature (see ref). However, none of them has identified a bethet order of a model to a distribution in which this study is trying to establish. In this study, the best order of the p and q are determined for every category of data distribution and level of the sample. Therefore, the objectives of this study are to look at the performance of the penalty functions of AIC, BIC HQIC, and FPE in ARIMA models order selection in relation to the type of the data being stationary and non-stationary and also being normal or non-normal in structure.

Autoregressive Moving Average (ARMA)

The autoregressive or moving average model becomes deficient when a higher order model with many parameters is needed to adequately describe the dynamic structure of a given data (Akeyede, 2016). The autoregressive Moving Average Model (ARMA) models was introduced by (Box and Tiao, 1975). ARMA model basically combines the techniques of Autoregressive and Moving Average Models into a compact form so that the number of parameters needed is minimized. If we assume that the series is partly Autoregressive and partly Moving Average, we obtained a quite general time series model (ARMA) as

$$X_t = \propto_1 X_{t-1} + \propto_2 X_{t-2} + \cdots + \propto_p X_{t-p} + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \cdots + \beta_q e_{t-q} + e_t \qquad (1)$$

We say that X_t, is an autoregressive and moving average of order p and q respectively. Symbolically, the model is represented by ARMA (p, q).

Autoregressive Integrated Moving Average (ARIMA)

A time series $\{W_t\}$ is said to follow an **Autoregressive Integrated Moving Average (ARIMA)** model if it d^{th} difference, $W_t = \nabla^d X_t$ is a stationary ARMA process. If $\{W_t\}$ follows an ARMA (p, q) model, we say that W_t

is an ARIMA (p,d,q) process (Akeyede, 2016). Fortunately, for practical purposes, d shouldn't exceed 2 (i.e we can usually take d = 1 or at most 2). (Cryer and Chan, 2008)

Then an ARIMA (p, 1, q) process is as follows:

$W_t = Y_t - Y_{t-1}$ we have

$$W_t = \emptyset_1 W_{t-1} + \emptyset_2 W_{t-2} + \cdots \emptyset_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \qquad (2)$$

or, in terms of the observed series we will have:

$$Y_t - Y_{t-1} = \emptyset_1(Y_{t-1} - Y_{t-2}) + \emptyset_2(Y_{t-2} - Y_{t-3}) + \cdots \emptyset_p(Y_{t-p} - Y_{t-p-1}) + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

which we may rewrite as

$$Y_t = (1 + \emptyset_1)Y_{t-1} + (\emptyset_2 - \emptyset_1)Y_{t-2} + (\emptyset_3 - \emptyset_2)Y_{t-3} + \cdots + (\emptyset_p - \emptyset_{p-1})Y_{t-p} - \emptyset_p(Y_{t-p-1}) + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

We call this the **difference equation form** of the model. Notice that it appears to be an ARMA (p + 1,q) process. However, the characteristic polynomial satisfies

$$1 - (1 + \emptyset_1)x - (\emptyset_2 - \emptyset_1)x^2 - (\emptyset_3 - \emptyset_2)x^3 - \cdots - (\emptyset_p - \emptyset_{p-1})x^p - \emptyset_p x^{p+1}$$

$$= \left(1 - \emptyset_1 x - \emptyset_2 x^2 - \cdots - \emptyset_p x^p\right)(1 - x) \qquad (3)$$

Which can be easily checked; this factorization clearly shows the root at x = 1, which implies non-stationarity. The remaining roots, however, are the roots of the characteristic polynomial of the *stationary* process ∇Y_t Explicit representations of the observed series in terms of either W_t or the white noise series underlying W_t are more difficult than in the stationary case.

MATERIALS AND METHODS

Data are simulated from normal and non-normal distribution at sample sizes of 20, 40, and 200. The situations were carried out 1000 times for every sample size and distribution to form 1000 iterations. At every iteration, the AIC, BIC, HQIC, and FPE for every other (p,d,q) of the ARIMA model, are computed and their average values are recorded in tables and plotted on graphs. A model with the lowest values of the criteria at a particular distribution and sample size is considered the best for that sample size and distribution. The stationarity assumptions of the absolute values of the sum of coefficients of the orders of autoregressive less than unity and white noise assumptions are observed so as to achieve the stationarity of the data generated while it is violated to obtain non-stationary data structures. The Effect of sample size and different underlined distributions of data when determining the order of the models were examined on each of the general ARMA and ARIMA model sections on the data simulated.

Models Considered for Simulation

Data is generated from second orders of Autoregressive functions given as follows:

Model AR (2): $Y_{ti} = 0.3Y_{ti-1} - 0.6Y_{ti-2} + e_t$,

$t = 1,2,\ldots,20, 40, 60, 80, 100, 120, 140, 160, 180\ and\ 200.\ i = 1,2,\ldots,1000$

The following codes were written to simulate data of sample size 50 from model 1 above

```
x <- e <- rnorm(20)
for (t in 3:20)x[t] <- 0.3*x[t-1]-0.6*x[t-2]+e[t]
```

For stationary cases, data will be simulated for both response variables and error terms from normal distribution with mean zero and variance one i.e;

$$Y_{ti} \sim N(0,1) and e_{ti} \sim N(0,1)$$

The normality of error term with zero mean and positive variance indicates that the error term is a white noise and therefore the data generated from these series is stationary.

Models Assessment Criteria

The criteria of AIC, BIC, HQIC and FPE were used to assessed the adequacy and efficacy of the study; presented as follows:

Akaike information criteria ((AIC), Akaike, 1969)

AIC (p, q)=log $[(\hat{\sigma}^2{}_{p,q}) + \frac{(p+q)2}{T}$

Schwarz Information Criteria (SIC), Schwarz, 1978)

SIC(p, q)=log $[(\hat{\sigma}^2{}_{p,q}) + \frac{(p+q)\log(T)}{T}$

Hannan-Quinn Information Criteria (HQIC), Hannan-Quinn, 1979)

HQIC(p, q)=log $[(\hat{\sigma}^2{}_{p,q}) + \frac{(p+q)2\log(\log(T))}{T}$

SIC and HQIC are more consistent than AIC (Hall, 1994).

Where p and q are the orders of the model, T number of observations/ sample size and $\hat{\sigma}^2{}_{p,q}$ mean square error (MSE) of the model

Stationarity

Stationarity is one most important assumption in time series model if meaningful inferences is to be made about the structure of a stochastic process on the basis of an observed records of such process. The basic idea of stationarity requirement is that the probability laws that guide the behavior of the stochastic process do not change with change in time. Implying that the process is in state of statistical equilibrium.

Specifically, a process $\{Yt\}$ is declared **strictly stationary** if the Joint Distributions of: $Y_{t1}, Y_{t2}, \ldots, Y_{tn}$ is the equivalent to the Joint Distributions of $Y_{t1-k}, Y_{t2-k}, \ldots, Y_{tn-k}$ for any choices of time points $t_1, t_2, \ldots, t_n$ and any choices of time lag k. Similarly, a stochastic process $\{Yt\}$ is considered to be **weakly** (or **second-order**) **stationary** if it satisfies the following:

1. its mean function is time invariant, and
2. $\gamma_{t,t-k} = \gamma_{0,k},\ for\ all\ time\ t\ at\ lag\ k$

The p^{th}-order autoregressive model:

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots \emptyset_p Y_{t-p} + e_t$$

stationary solution to the equation exists if and only if the p roots of the AR characteristic equation each exceed 1 in absolute value

That is, for the roots to be greater than 1 in absolute value, it is necessary, but not sufficient, that both:

$$\emptyset_1 + \emptyset_2 + \cdots + \emptyset_p < 1$$

$$|\emptyset_p| < 1$$

Evaluation, Comparison and Preference of model

At each scenario of specification, the best order and sample size, the models were examined and compared using Akaike information criteria (AIC), Bayesian information criteria (BIC), Hannan-Quin information criteria (HQIC) and Final Prediction Error (FPE) criteria. The model with minimum criteria under different scenario of simulation was taken as the best

RESULTS AND DISCUSSION

The analyses of the ARIMA (p,d,q) models are presented in tables 1- and plotted on figure 1- as follows and the discussion of every category is discussed under each graph.

Table 1: AIC and BIC Values of ARIMA (p, d, q) Model for Stationary Normal Data

	AIC				BIC			
Sample Sizes	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**
20	74.8945	63.8765	60.9994	48.863	77.72788	67.65432	64.7771	53.58519
40	162.805	113.897	118.760	129.305	167.796	120.5515	125.415	137.6228
60	246.016	190.510	172.145	185.221	252.2487	198.8206	180.455	195.6086
80	301.299	273.475	236.545	221.620	308.4074	282.953	246.023	233.4675
100	329.256	346.102	293.435	287.795	337.0414	356.4832	303.815	300.7712
120	443.988	386.100	354.834	370.237	452.3258	397.217	365.950	384.133
140	457.121	476.512	387.357	369.926	465.9253	488.2499	399.095	384.5984
160	506.509	491.124	477.409	474.522	515.7167	503.4004	489.684	489.8671
180	600.517	565.365	518.497	540.780	610.0801	578.1146	531.247	556.7178
200	607.984	606.226	586.645	544.579	617.8643	619.3999	599.818	561.0455

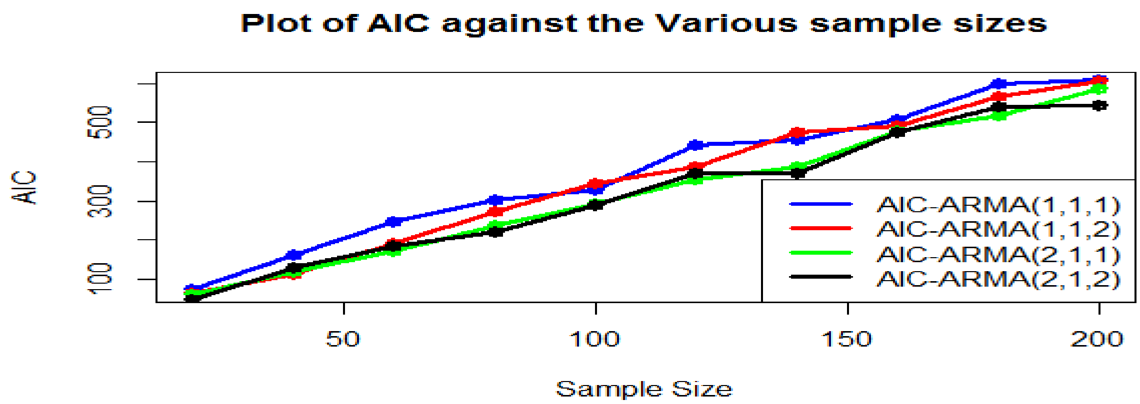

Figure 1a: AIC values for ARIMA (p, d, q) models form Normal Data

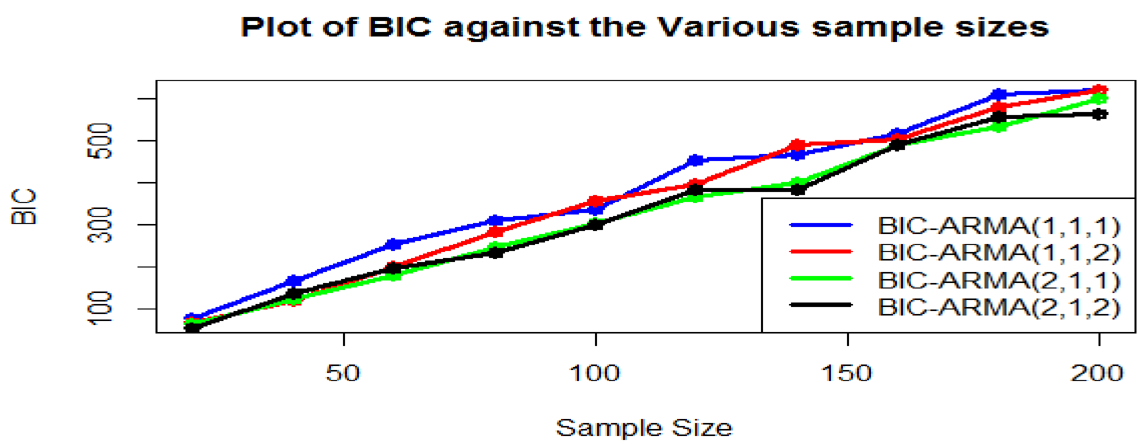

Figure 1b: BIC values for ARIMA (p, d, q) models from Normal Data

Table 1 shows the relative performance of r the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC for each model are recorded in the table above and then plotted in figures 1a and 1b respectively. Both AIC and BIC increase with increase in sample sizes for the simulated data; but decreases with increase in order. At sample size 20 ARIMA (2, 1, 2) was the best by both AIC and BIC, at 40 ARIMA (1,1,2) was picked as the best. ARIMA (2,1,1) was picked as the best for sample size 60, at 80 and 100 ARIMA (2,1,2) were selected as the best models, ARIMA (2,1,1) were the best for 120 and 180, ARIMA (2,1,2) were picked as the best at 140, 160 and 200 respectively.

Table 1: HQIC and FPE Values of ARIMA (p, d, q) Model for Stationary Normal Data

	HQIC				FPE			
Sample Sizes	ARIMA(1 ,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARMA (2,1,2)
20	65.6687	79.7087	63.6031	62.8375	37.44889	46.02889	38.5723	30.545
40	134.421	118.821	107.831	104.282	71.4	62.77895	58.1081	53.45778
60	189.598	184.678	163.237	154.156	98.32345	106.3834	85.5363	81.64571
80	264.67	238.61	216.225	208.540	136.0149	122.3167	111.759	109.7663
100	318.088	311.228	301.303	286.177	162.3569	158.7869	155.105	148.395
120	436.564	374.624	352.076	350.028	222.4432	190.4234	180.126	176.9914
140	501.830	427.430	413.426	407.141	254.9003	216.622	206.420	208.7788
160	567.697	508.557	463.006	435.654	287.7038	257.3852	235.291	237.9367
180	587.509	568.969	518.383	508.718	296.9872	287.5089	262.868	259.0323
200	636.529	652.389	593.304	568.399	321.2922	329.3824	300.532	288.8578

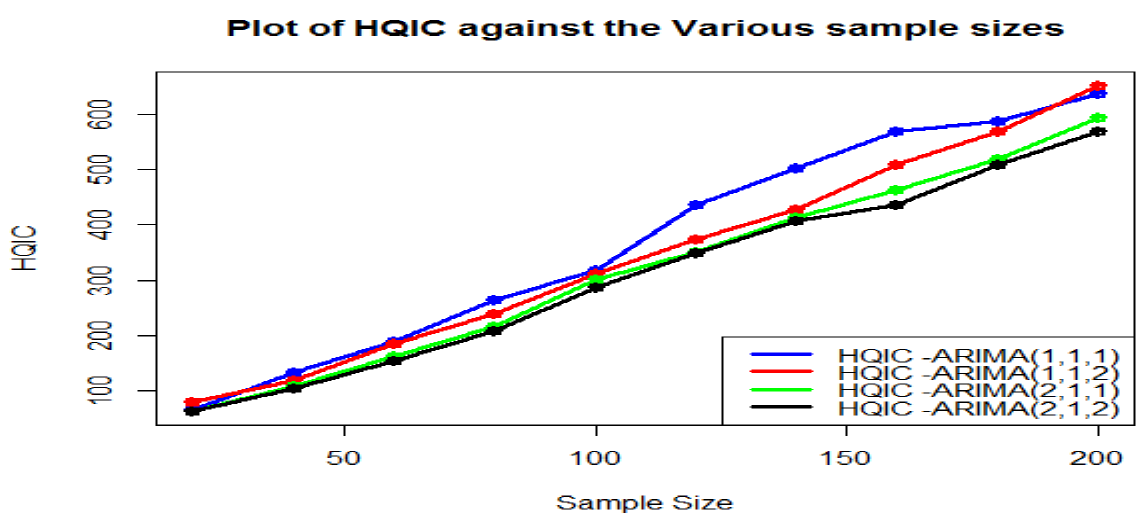

Figure 1c: HQIC values for ARIMA (p, d, q) models form Normal Data

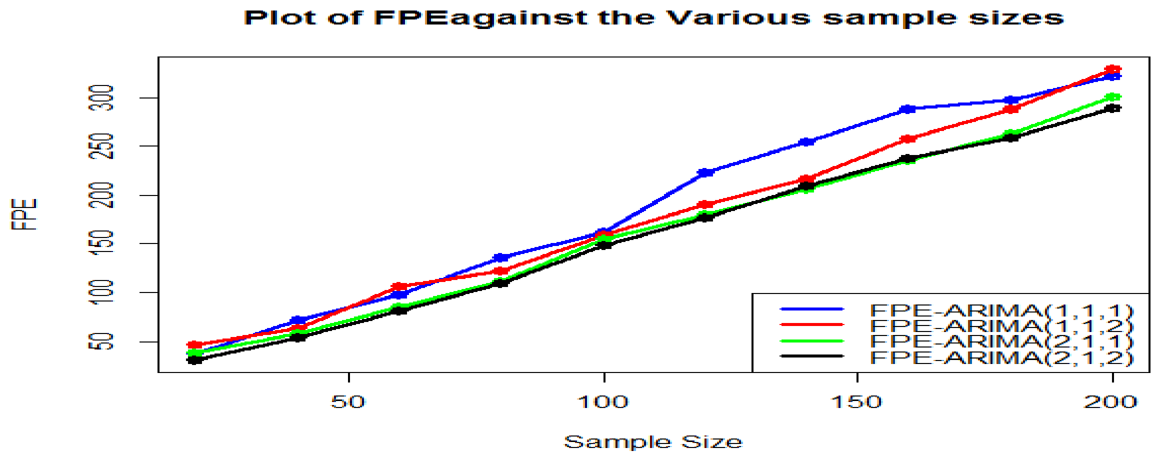

Figure 1d: FPE values for ARIMA (p, d, q) models form Normal Data

It was observed similarly from the table above that both the average values of HQIC and FPE increase with increase in sample size for the simulated data; but decrease with increase in the order of the models. The relative performances of the four fitted models are recorded in the table above and then plotted in figures 1c and 1d respectively. At sample size 20 model ARIMA (2, 1, 2) was the best picked by both HQIC and FPE, at 40 ARIMA (1, 1,2) was picked. ARIMA (2,1,1) was picked for sample size 60, at 80 and 100 ARIMA (2,1,2) were selected as the best, ARIMA (2,1,1) was selected for 120 and 180 and ARIMA (2,1,2) was picked as the beast at 140, 160 and 200 respectively.

Table 2: AIC and BIC Values of ARIMA (p, q) Model for Stationary Non-Normal Data (Exponential distribution)

	AIC				**BIC**			
Sample Sizes	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**
20	176.706	174.317	182.526	209.038	179.5396	178.0948	186.304	213.761
40	425.799	365.405	357.906	435.987	430.7897	372.0595	364.560	444.3048
60	648.552	609.798	560.666	595.870	654.785	618.1086	568.976	606.2582
80	897.393	837.343	762.807	793.582	904.502	846.8208	772.284	805.4295
100	1001.40	1010.34	1013.98	958.784	1009.195	1020.729	1024.36	971.7596
120	1290.16	1232.07	1198.21	1232.80	1298.502	1243.191	1209.33	1246.699
140	1470.28	1402.76	1370.90	1345.74	1479.089	1414.503	1382.64	1360.416
160	1666.12	1678.7	1542.92	1573.34	1675.328	1690.975	1555.19	1588.685
180	1820.79	1769.84	1828.64	1732.16	1830.361	1782.59	1841.39	1748.097
200	2062.35	1928.01	1891.70	1952.03	2072.233	1941.185	1904.88	1968.499

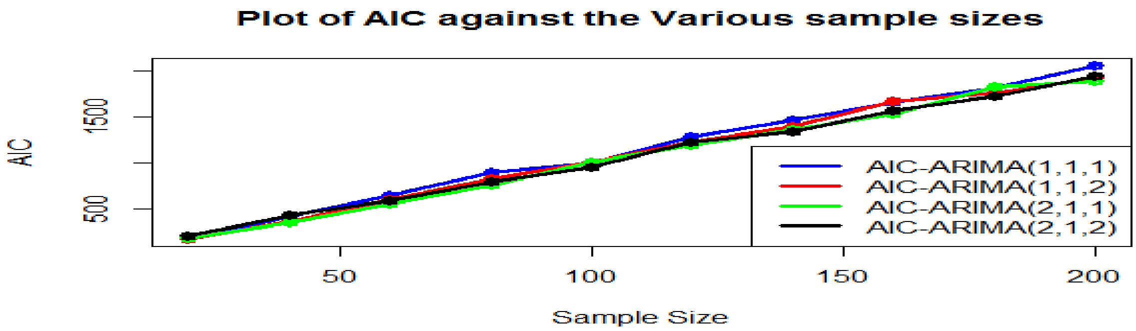

Figure 2a: AIC values for ARIMA (p, d, q) Models form a Non-normal Data

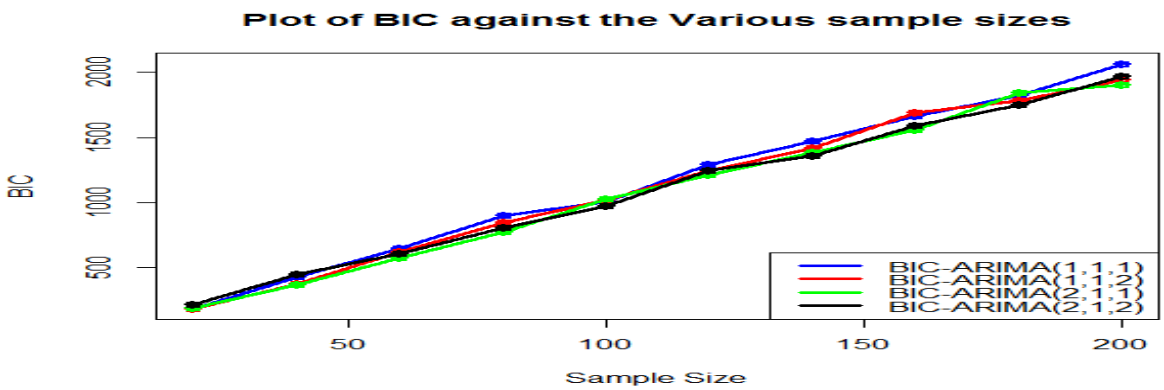

Figure 2b: BIC values for ARIMA (p, d, q) Models form a Non-normal Data

The Table shows the relative performance of the four fitted models on simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC for each of the model are recorded in the table above and then plotted in figures 2a and 2b respectively. At sample sizes 20 both AIC and BIC picked ARIMA (1, 1,2) as the best, model, at 40, 60, 80, 120, 160 and 200 they picked ARIMA (2,1,1) as the best. ARIMA (2,1,2) model was as the best picked for sample sizes 100, 140, and 180 respectively.

Table 2: HQIC and FPE Values of ARIMA (p) Model for Stationary Non-Normal Data (Exponential distribution)

	HQIC				FPE			
Sample Sizes	**ARIMA (1,1,1,**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**	**ARIMA (1,1,1,**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**
20	152.2688	140.7831	143.1031	141.5175	90.37111	90.2353	92.35176	99.555
40	337.2213	280.9719	276.2119	334.3026	183.4737	158.7165	190.8154	197.9144
60	477.9384	479.3176	435.2176	445.0169	252.4362	260.2121	242.42	247.8514
80	610.75	601.6651	579.3451	598.2001	317.9287	319.4961	319.0249	324.0521
100	834.2687	735.9431	736.0431	726.8974	430.9812	385.8677	385.9208	383.1183
120	990.184	930.176	880.176	883.0281	508.6366	483.9997	462.9741	465.2672
140	1172.651	1059.286	1043.286	979.2616	600.0323	547.8361	539.4858	511.6659
160	1231.997	1267.346	1148.686	1158.715	628.2627	652.8306	653.5262	662.2374
180	1439.269	1433.004	1339.924	1330.918	732.4375	735.6807	687.5631	684.5005
200	1571.83	1621.084	1493.924	1506.359	798.3897	830.0742	764.5578	776.9798

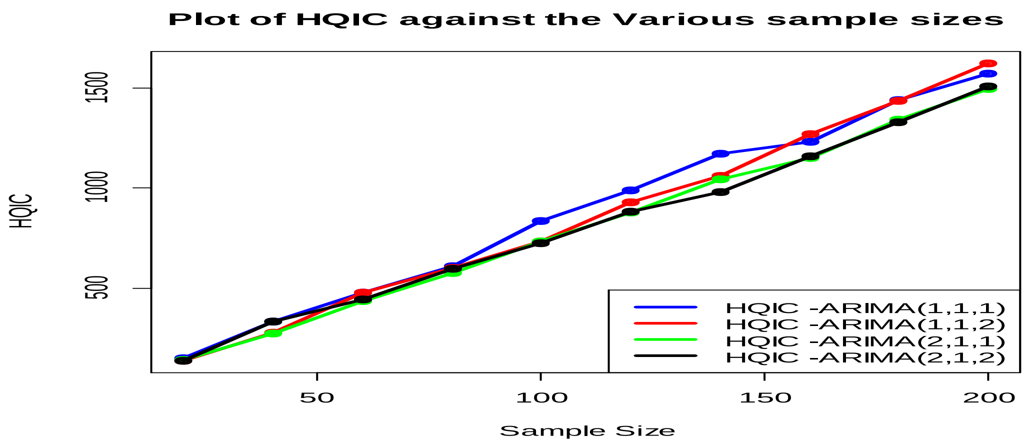

Figure 2c: HQIC values for ARIMA (p, d, q) Models form a Non-normal Data

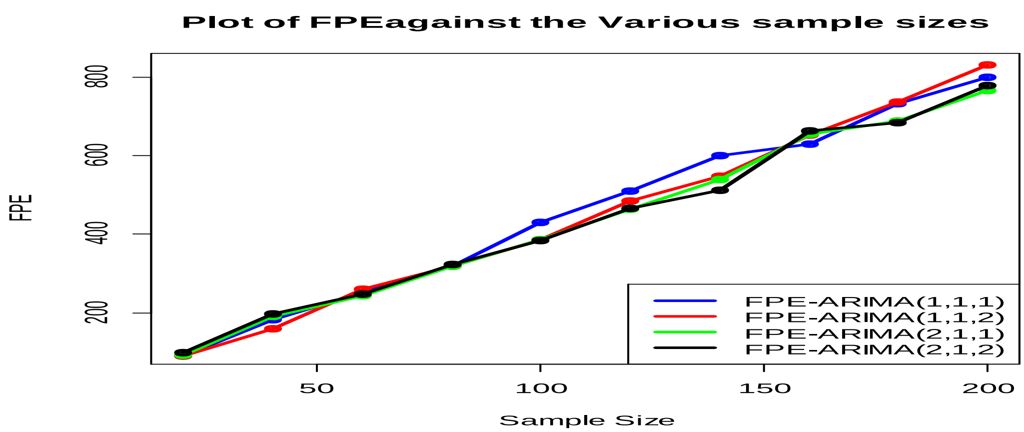

Figure 2d: FPE values for ARIMA (p, d, q) Models form a Non-normal Data

Similarly, the Table above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of HQIC and FPE for each of the models are recorded in the table above and then plotted in figures 2c and 2d respectively. At sample sizes 20 both AIC and BIC picked ARIMA (1, 1,2) as the best, model, at 40, 60, 80, 120, 160 and 200 they picked ARIMA (2,1,1) as the best. ARIMA (2,1,2) model was as the best picked for sample sizes 100, 140, and 180 respectively.

Table 3: AIC and BIC Values of ARIMA (p) Model for Uniform Stationary Data

	AIC				BIC			
Sample Sizes	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**
20	102.756	108.535	110.195	108.164	105.59	108.4151	113.973	112.887
40	202.937	188.239	212.130	187.926	207.9285	194.8939	218.785	196.2447
60	292.051	278.122	290.181	274.508	298.2836	286.4323	298.491	284.8965
80	373.930	360.644	358.135	357.878	381.0387	370.1222	367.613	369.7259
100	463.040	458.664	448.306	441.796	470.8255	469.0448	458.686	454.7721
120	570.121	521.541	543.579	537.352	578.4584	532.6576	554.696	551.2484
140	614.511	597.187	624.306	584.610	623.3148	608.9254	636.044	599.2827
160	726.655	712.008	712.051	681.150	735.8625	724.2845	724.326	696.4951
180	822.195	765.903	782.957	729.186	831.758	778.6526	795.707	745.1232
200	898.535	850.923	861.888	853.599	908.4151	864.097	875.061	870.0656

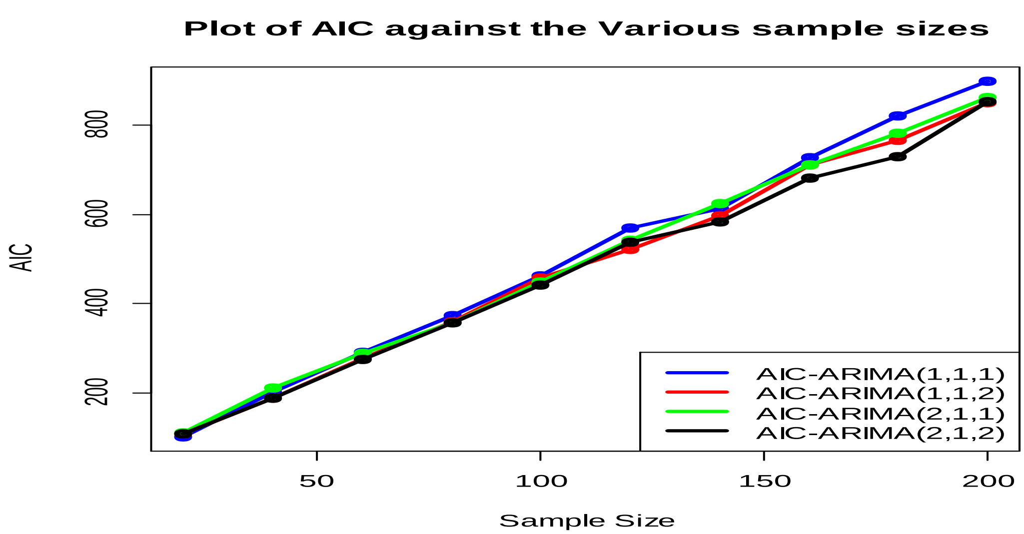

Figure 3a: AIC values for ARIMA (p, d, q) Models from a Non-normal Data

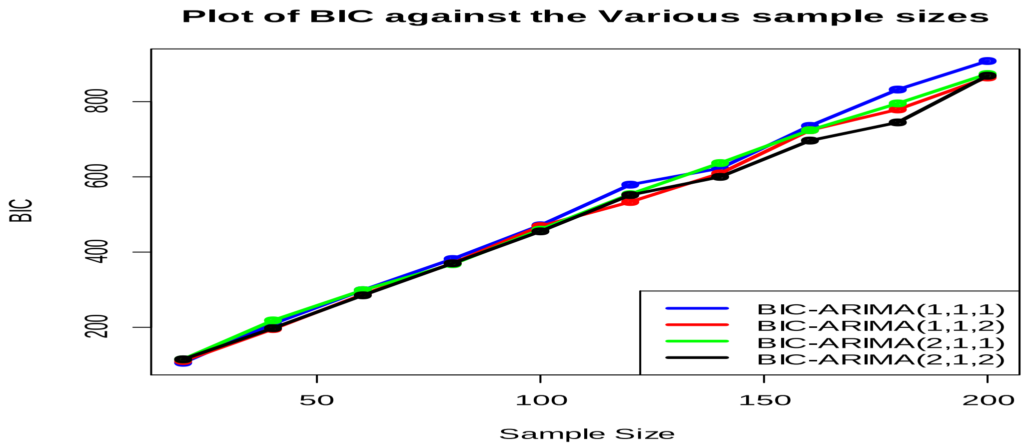

Figure 3b: BIC values for ARIMA (p, d, q) Models from a Non-normal Data

Table 3 above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC for each of the model are recorded in the table above and then plotted in figures 3a and 3b respectively. At sample sizes 20, ARIMA (1,1,1) was selected as the best fit. ARMA (2, 1, 2) was the best fit at sample sizes 40, 60, 80,140, 160, 180 and 200 respectively. While ARIMA (1, 1, 2) was the best fit at 120

Table 3: HQIC and FPE Values of ARIMA (p) Model for Stationary Uniform Non-normal Data

	HQIC				FPE			
Sample Sizes	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**	**ARIMA (1,1,1)**	**ARIMA (1,1,2)**	**ARIMA (2,1,1)**	**ARIMA (2,1,2)**
20	98.4687	102.163	109.063	92.5975	53.64235	64.65706	69.3247	56.70176
40	196.121	192.831	192.431	190.682	126.1213	107.5	107.267	102.3827
60	267.178	277.857	287.237	273.896	144.5353	148.8789	154.062	145.1321
80	391.59	357.765	352.845	366.100	207.8665	188.0435	185.391	190.9431
100	478.188	452.143	455.803	463.097	250.6404	235.1904	237.133	239.3847
120	556.144	532.496	575.216	541.568	289.0395	274.9628	297.418	287.5723
140	640.450	616.526	616.186	620.641	330.9145	316.7607	316.583	317.2408
160	723.697	696.066	702.806	697.454	372.3045	356.2744	359.773	355.3089
180	806.649	767.843	772.523	780.898	413.5903	391.8268	394.246	396.8722
200	888.189	854.864	857.964	815.899	454.1842	435.2959	436.893	413.5017

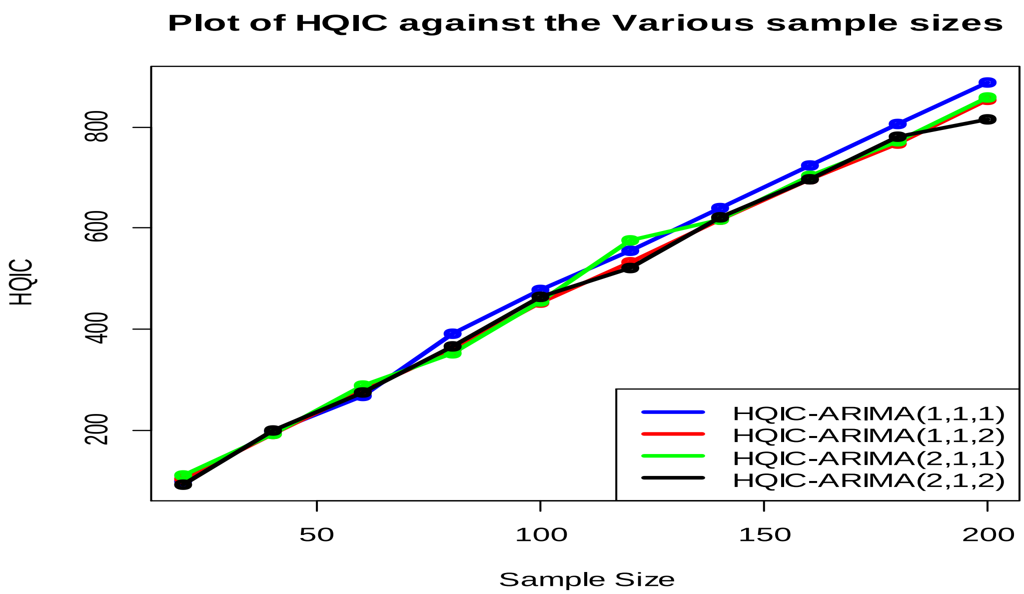

Figure 3c: HQIC values for ARIMA (p, d, q) Models from a Non-normal Data

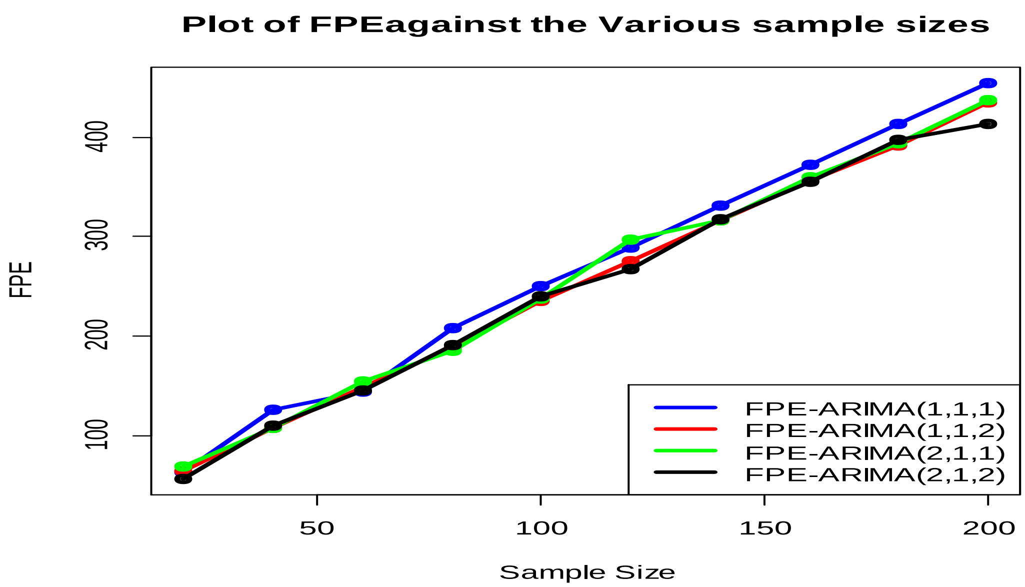

Figure 3d: FPE values for ARIMA (p, d, q) Models from a Non-normal Data

Similarly, table 3 above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of HQIC and FPE are recorded and then plotted in figures 3c and 3d. At sample sizes 20, ARIMA (1, 1, 1) was selected as the best fit. ARMA (2, 1, 2) was the best fit at sample sizes 40, 60, 80,140, 160, 180 and 200 respectively. While ARIMA (1, 1, 2) was the best fit at 120

Table 4: AIC and BIC Values of ARIMA (p) Model for Non-Stationary Data from Normal distribution

	AIC				BIC			
Sample Sizes	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**
20	140.364	130.160	127.066	137.136	143.198	133.938	130.844	141.8587
40	255.331	240.914	233.835	239.384	260.3224	247.5683	240.489	247.7023
60	393.303	356.778	360.112	353.028	399.5357	365.0889	368.422	363.4158
80	492.361	469.736	446.010	450.587	499.4699	479.2144	455.488	462.4345
100	627.858	581.449	564.311	554.318	635.6438	591.8303	574.691	567.2944
120	747.513	696.462	650.353	683.09	755.8512	707.5794	661.470	696.9856
140	840.989	811.896	830.548	810.532	849.7925	823.6345	842.286	825.2046
160	993.802	898.386	912.704	901.088	1003.009	861.3004	924.979	916.4332
180	1076.67	1014.99	1003.49	1021.39	1086.239	1027.747	1016.24	1037.332
200	1172.33	1118.48	1126.40	1862.39	1182.21	1131.658	1191.51	1142.873

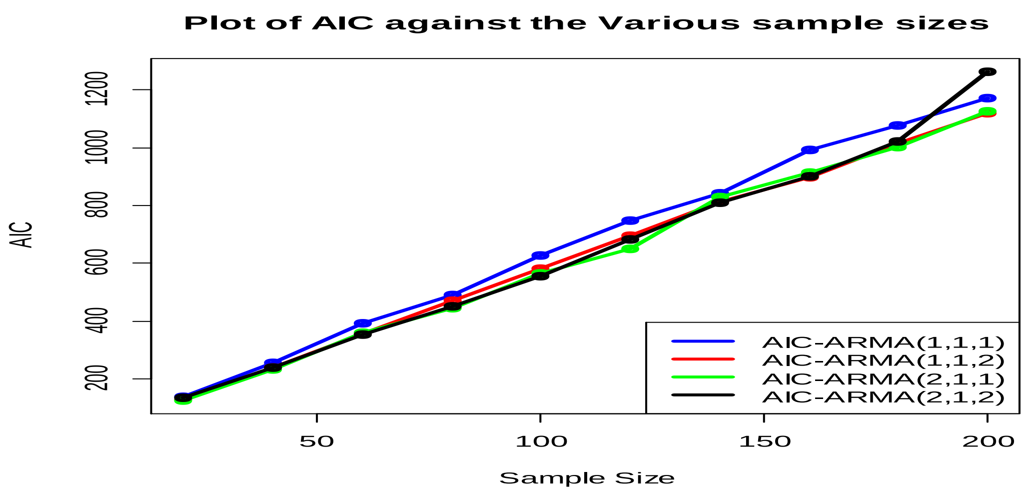

Figure 4a: AIC values for ARIMA (p, d, q) Models from a Non-normal Data

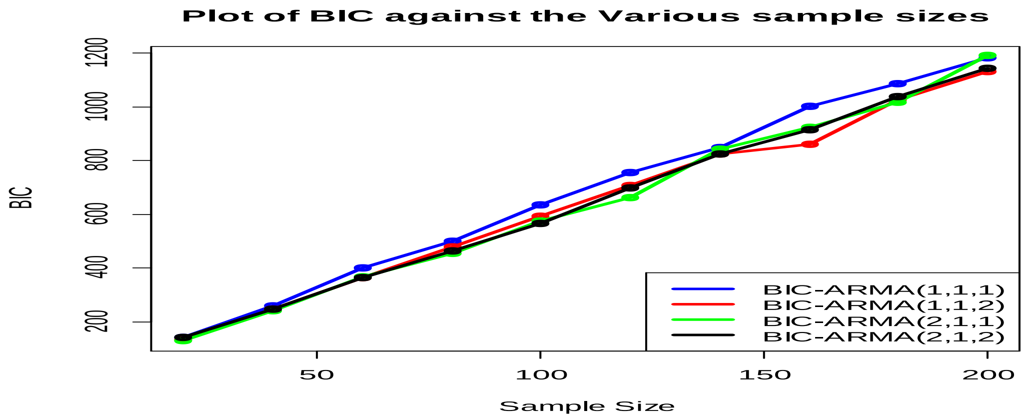

Figure 4b: BIC values for ARIMA (p, d, q) Models from a Non-normal Data

Table 4. above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC for each model are recorded in the table above and then plotted in figures 4.a and 4b respectively. It was observed that the values of both AIC and BIC increases with increasing sample sizes of the simulated data. At sample sizes 20, 40, 80, 120 and 180 ARIMA (2,1,1) was selected as the best fit. ARIMA (1, 1, 2) was the best fit at sample sizes160 and 200. While at sample sizes 60, 100 and 140 ARIMA (2, 1, 2) was the best fit respectively

Table 4: HQIC and FPE Values of ARIMA (p) Model for Non-Stationary Data from Normal distribution

	HQIC				FPE			
Sample Sizes	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**
20	138.748	128.743	125.643	135.9175	82.10889	82.63765	80.5405	95.355
40	254.561	240.751	233.671	239.8226	137.7932	135.3454	131.231	140.1767
60	392.938	357.237	360.577	354.2969	207.0052	192.7468	194.592	196.0114
80	492.27	470.605	446.885	452.4001	255.6508	248.8599	236.075	243.4784
100	627.968	582.603	565.483	556.5374	323.621	304.4553	295.365	294.84
120	747.784	697.856	651.756	685.6281	383.3281	361.8828	337.650	359.7603
140	841.370	813.486	832.126	813.3216	429.5912	419.5536	429.281	423.8153
160	994.297	896.639	914.446	904.0749	506.4038	508.9146	469.637	468.3882
180	1077.26	1016.88	1005.38	1024.578	547.3701	520.5678	514.622	528.6864
200	1173.01	1120.48	1180.34	1129.578	594.9512	572.1509	602.992	583.5727

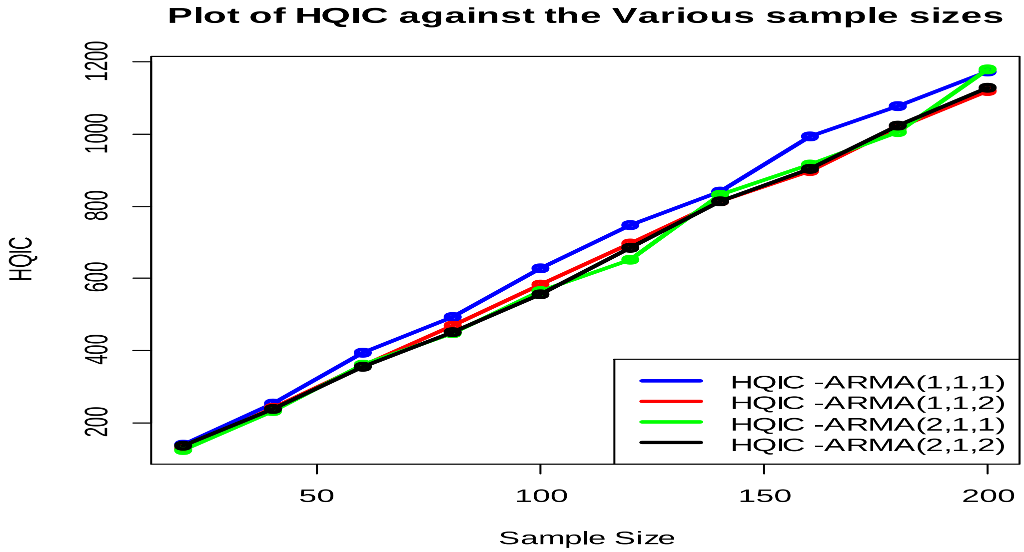

Figure 4c: BIC values for ARIMA (p, d, q) Models from a Non-normal Data

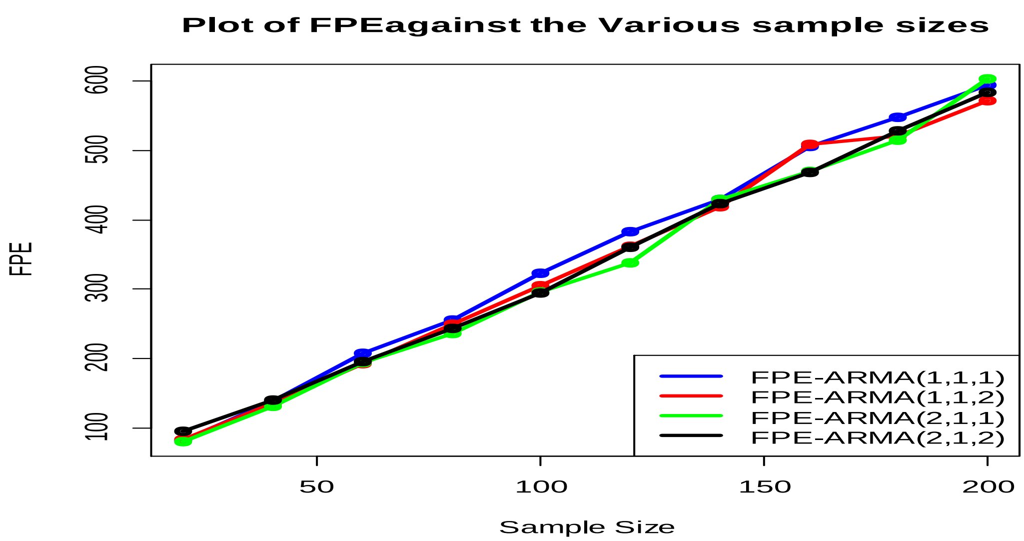

Figure 4d: BIC values for ARIMA (p, d, q) Models from a Non-normal Data

Similarly, the table above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of HQIC and FPE are recorded in the table above and plotted in figures 4c and 4d respectively. It was observed that the values of both HQIC and FPE increases with increase in sample sizes of the simulated data. At sample sizes 20, 40, 80, 120 and 180 ARIMA (2, 1,1) was selected as the best fit. ARIMA (1, 1,2) was the best fit at sample sizes160 and 200. While at sample sizes 60, 100 and 140 ARIMA (2,1,2) was the best fit respectively.

Table 5: AIC and BIC Values of ARIMA (p) Model for Non-Stationary Data from exponential distribution

	AIC					BIC		
Sample Sizes	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**
20	197.686	191.440	195.023	195.162	200.5196	195.2184	198.801	199.8845
40	393.476	369.337	401.665	401.779	398.4676	375.9919	408.32	410.097
60	585.7	552.005	636.099	561.930	591.9327	560.3152	644.409	572.3185
80	776.789	791.514	815.798	793.706	783.8978	800.9925	825.275	805.5538
100	981.896	991.670	996.828	991.304	989.6818	1002.051	1007.20	1004.28
120	1190.92	1171.79	1229.73	1178.32	1199.261	1182.907	1240.85	1192.221
140	1394.64	1347.05	1390.95	1367.75	1403.445	1358.797	1402.69	1382.427
160	1583.81	1594.5	1592.70	1543.15	1593.024	1606.776	1717.90	1558.498
180	1759.24	1722.50	1769.28	1785.05	1768.809	1735.252	1782.03	1800.988
200	2029.57	1987.58	2002.85	1962.51	2039.457	2000.763	2016.03	1978.977

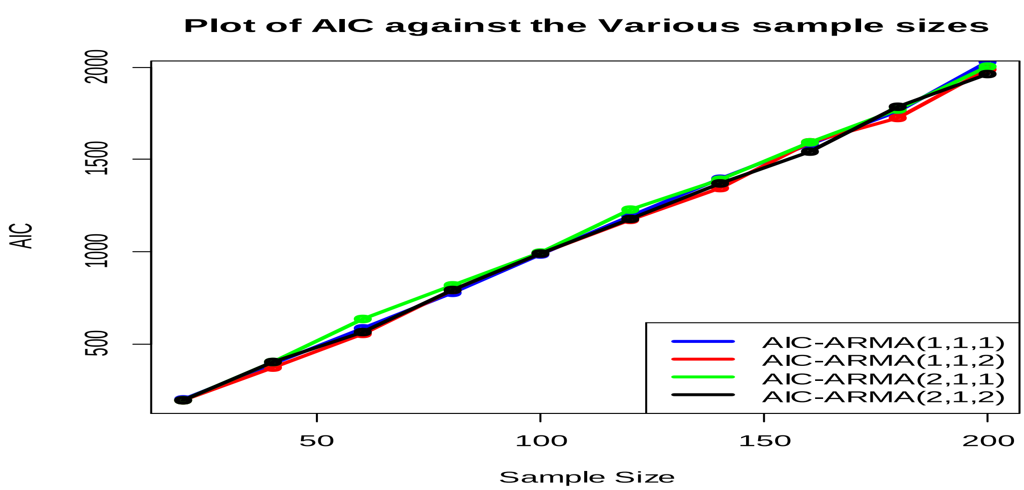

Figure 5a: AIC values for ARIMA (p, d, q) Models from a Non-normal Data

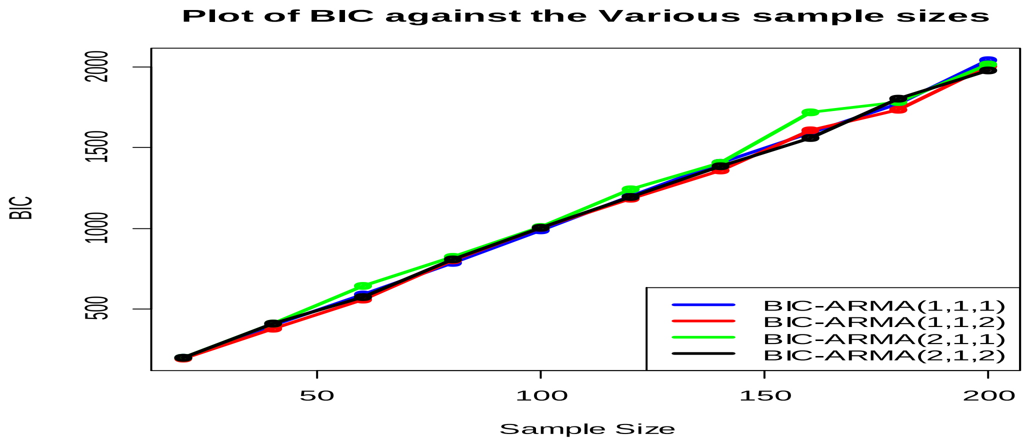

Figure 5b: AIC values for ARIMA (p, d, q) Models from a Non-normal Data

Table 5 above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC are recorded in the table above and then plotted in figures 5a and 5b respectively. It was observed that the values of both AIC and BIC increases with increasing sample sizes of the simulated data. At sample sizes 20, 40, 60, 120, 140 and 180 ARIMA (1, 1, 2) was selected as the best fit. ARIMA (1, 1, 1) was the best fit at sample sizes 80 and 100 respectively. While at sample sizes 160 and 200 ARIMA (2, 1, 2) was the best fit

Table 5: HQIC and FPE Values of ARIMA (p) Model for Non-Stationary Data from exponential distribution

	HQIC				FPE			
Sample Sizes	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**	**ARMA (1,1,1)**	**ARMA (1,1,2)**	**ARMA (2,1,1)**	**ARMA (2,1,2)**
20	196.0688	187.8288	193.6031	191.7431	117.1378	112.1022	126.5135	138.87
40	392.7013	366.5613	401.4919	402.2226	214.1337	209.9678	228.7484	239.4211
60	585.3384	549.6384	636.557	563.2169	309.8397	300.6316	347.1079	315.3943
80	776.69	792.3851	816.6651	795.5201	405.1536	422.2868	435.3727	433.0974
100	982.0087	992.8431	997.9831	993.5174	507.8663	522.2631	524.9921	531.5375
120	1191.184	1173.196	1231.136	1180.848	612.5434	611.741	642.1967	624.4469
140	1395.031	1348.646	1392.546	1370.542	714.4452	698.8525	721.7638	718.8141
160	1584.317	1596.246	1590.959	1546.155	808.8824	823.5653	802.697	805.8918
180	1759.829	1724.384	1771.164	1788.238	896.3193	886.3093	910.4922	927.8723
200	2030.25	1989.584	2004.864	1965.859	1032.23	1019.936	1027.809	1016.107

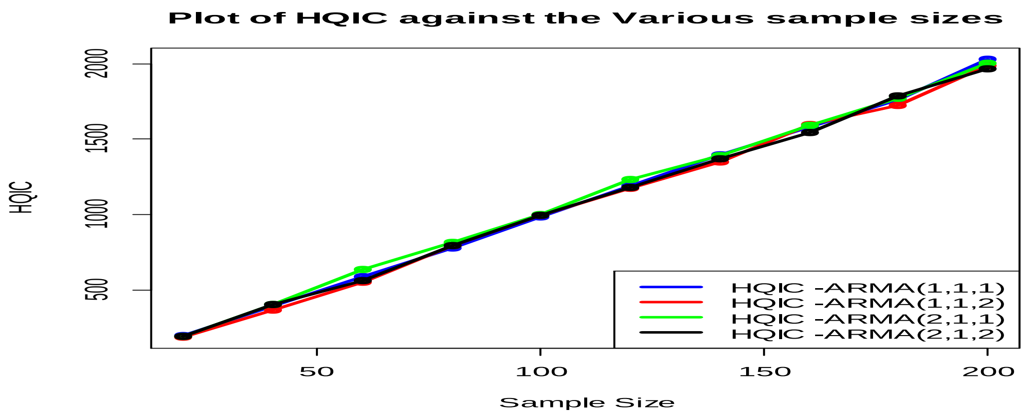

Figure 5c: HQIC values for ARIMA (p, d, q) Models from a Non-normal Data

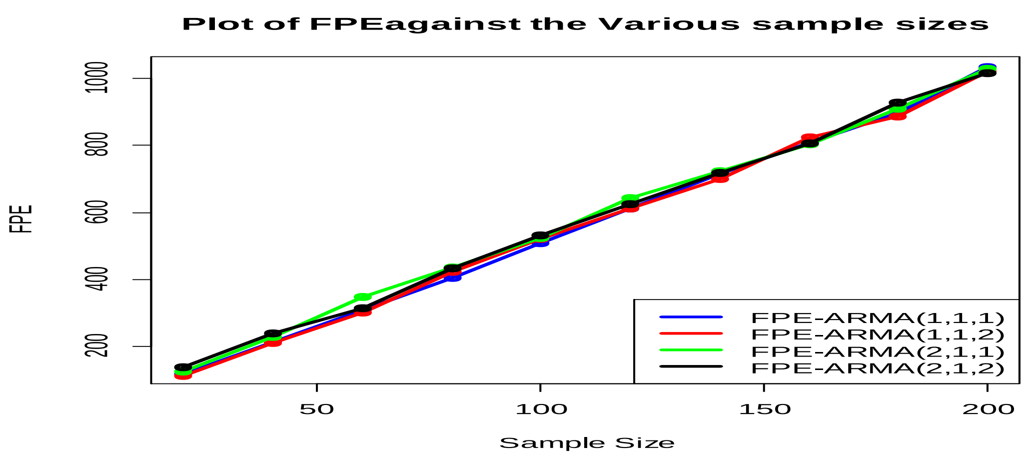

Figure 5d: FPE values for ARIMA (p, d, q) Models from a Non-Normal Data

Similarly, Table above shows the relative performance of the four fitted models on the simulated data with 1000 iterations at various sample sizes. The average values of AIC and BIC are recorded in the table above and then plotted in figures 5c and 5 d respectively. It was observed that the values of both AIC and BIC increases with increasing sample sizes of the simulated data. At sample sizes 20, 40, 60, 120, 140 and 180 ARIMA (1, 1, 2) was selected as the best fit. ARIMA (1, 1, 1) was the best fit at sample sizes 80 and 100 respectively. While at sample sizes 160 and 200 ARIMA (2, 1, 2) was the best fit

CONCLUSION

The best orders for the ARIMA at different sample sizes were assessed. It was observed that for the ARMA the order stocked to the principle of parsimony i.e., models with lower order selected at most of the sample sizes considered. At the middle to the larger sample (100 to 200) the models have close performance. Similarly, the effect of different levels of parameters (0.3, 0.6, and -0.3, -0.6) at the sample size of 20, 40, 60, 80, 100, 120, 140, 160, 180 and 200 which represent to small, moderate and large sample sizes respectively on the simulated data from the stationary normal and non-normal data. Therefore, we conclude in general that the selections of the order for the models considered in this study are tied more to the underlying distribution of the series in relation to the Stationarity and non-Stationarity of the series as it will lead to identification of proper model. Since the selection are almost identical in the stationary and non-stationary from both normal and non-normal data structure but varies with the variation in the distribution of the series. The need to develop a methodology for model selection that combines both objective and subjective techniques is strongly recommended.

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