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Research Article

An Analytical Formula for $1s\,2s2p\,3s3p3d$ Energy Shifts of Finite-Size Nuclei due to Fluctuating Vacuum Fields

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ABSTRACT

In quantum electrodynamics, the vacuum is not empty but has fluctuating fields which cause an orbiting lepton (electron or muon) to deviate from its quantum orbit and as a result, weaken its coupling with the nucleus, thus causing small changes in lepton energy states. Therefore, in this study, the quantum electrodynamics theory is then applied to determine qualitatively the mean square positions of an oscillating lepton by taking into account the vacuum fields and nuclear size effects. The first-order time-independent perturbation theory as an approximation method is applied to solve for the small changes in the nuclear-lepton interaction caused by the two effects. The general expression that can be applied to calculate the changes in lepton energy states is deduced and then applied to calculate numerically these effects on the energy states of single-electron and single-muonic atoms. The results showed the dependence of these effects on the principal quantum number n, the orbital quantum number l, the azimuthal quantum number m and the proton number Z. A more significant effect on s (l = 0) states of the large atomic nucleus and muonic atoms is observed. The obtained energy level corrections can be considered as additional energy level corrections that have to be investigated for reanalyzing the existing measured nuclear structure effects and for comparison with future spectroscopic investigations. Moreover, the new proposed formula would be suitable for calculating the energy level corrections in both muonic and electronic atoms caused by the fluctuating vacuum fields.

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INTRODUCTION

An atomic nucleus with nucleon number A is characterized by nuclear size, $R = r_0A^{1/3}$, with r_0 being constant. This is a clear indication that isotopes could have different nuclear charge distributions and thus different energy states. This difference in energy states and the isotope shifts, which are predicted theoretically and observed experimentally, provides clear evidence for the finite-size nature of atomic nucleus. This contradicts the description of the atomic nucleus as being a point-charge having $1/r$ lepton-nuclear interaction. The the atomic nucleus is not truly point-charge (as described from the solution of Schrödinger or Dirac equation), rather has a finite-size charge distribution with some shape over which the proton charge is distributed (Palffy, 2011; Adamu and Ngadda, 2014; Adamu *et al*., 2018).

The description of the atomic nucleus as being a finite size revealed other nuclear structures (for example nuclear radius) (Sirma *et al*., 2020), spectral investigation (for example, hyperfine splittings and Lamb shift) and in solving the proton charge radius that puzzle scientist for decades (Faustov *et al*., 2014). Furthermore, Adamu and Ngadda, (2018) suggested that the nuclear radius can be determined based on its potential charge distribution and showed that the potential charge of the nucleus has exceeded the nucleon size by a factor √3.

Another correction to point-charge $1/r$ nuclear potential is the deviation of orbiting lepton (electron or muon) from its quantum orbit as it interacts with fluctuating vacuum fields. This weakens the coupling of lepton with the nucleus and as a result affects its energy states (Borie and Rinker, 1982; Borie, 2014; Dalibard *et al.*, 1982; Deck *et al*., 2005; Dyson, 2006; El Shabshiry *et al*., 2013; Erickson, 1977; Vaks, 1959; Venkataram, 2013). The root-mean-square amplitude of the oscillation of electrons due to fluctuating vacuum fields was numerically calculated as about 1000^{th} of the size of the electron orbit (Adamu *et al*., 2019). An investigation on spectral changes due to nuclear size and vacuum fields effects (see, for example (Adamu and Ngadda, 2014; Adamu and Ngadda, 2015; Palffy, 2011; El Shabshiry *et al*., 2015) showed that such corrections are very small and can be ignored for light nucleus and on *p, d, f*, … energy states and that the corrections are more important for higher atomic nucleus and muonic atoms. Being 207 times heavier than an electron ($m_\mu = 207\ m_e$), and a distinctly small atomic orbitals radii, muonic atoms have an enhanced sensitivity to nuclear structure effects (Kanda, 2022; Godunov and Vysotsky, 2013; Antognini, 2015; Dong *et al*., 2011; Toth *et al*., 2021; Antognini *et al*., 2022). Other nuclear parameters such as root-mean-square radii, electric quadrupole and magnetic dipole moments were also extracted based on theoretical predictions and experimental measurements of the level structure and the transition energies in muonic atoms (Patoary and Oreshkina, 2018; Faustov *et al*., 2019).

Moreover, the analysis of spectroscopic x-ray measurements using muonic transitions revealed some anomalies and disagreements with previous theoretical predictions (Kardaras and Kosmas, 2015). The current experimental program to study the fine and hyperfine structure of the simplest muonic atoms is successfully measured in muonic hydrogen. The studies of the nucleon and nuclear structure based on light muonic atoms offer a new leap in precise determinations of the charge radii of the proton (Ji et al., 2016; Antognini *et al*., 2021) which disagreed with the previous determinations from the conventional methods of hydrogen spectroscopy and electron-proton scattering (Lensky *et al.*, 2022). The understanding of this strong dependence of the muonic atoms

on nuclear parameters, and the information about atomic nuclei that they can deliver, has triggered interest in the precise knowledge of the level structure of muonic atoms (Oreshkina, 2022). Therefore, in this work, we adopt the assumption of a finite – size model of the atomic nucleus and consider the effect of fluctuating vacuum fields on orbiting leptons, and then applied time-independent perturbation theory, as an approximation method to determine the resulting lepton-nuclear interactions and the subsequent energy levels changes.

METHODOLOGY

Based on the fact that the atomic nucleus is not a point-charge and that leptons interacting with the nucleus feel its finite-size and vacuum field's effects, a simple expression that could generally describe these effects on single-electron – muon atomic nuclei is formulated by applying the first-order time-independent perturbation.

The Average Potential for Perturbed Electron

As the result of its interaction with fluctuating vacuum fields, lepton deviates from its quantum orbit by an amount as shown in Figure 1. For such a small deviation, the average potential energy of the electron can be simplified using the Taylor expansion as:

$$\langle U(r+\delta\vec{r})\rangle = \langle U(r)\rangle + \langle \delta r\rangle \cdot \vec{\nabla} U(r) + \frac{1}{2}\langle (\delta\vec{r}\cdot\vec{\nabla})^2\rangle U(r) + \cdots = \langle U(r)\rangle + U'(r)$$

where

$$U'(R) = \frac{1}{6}\langle \delta\vec{r}^2\rangle_t \vec{\nabla}^2 U(R) \qquad (1)$$

is the perturbation due to vacuum fields fluctuations (Weisskopf, 1949) and the Laplacian of the potential $\vec{\nabla}^2 U(R)$, is proportional to the extended nuclear charge distribution:

$$U(R) = -\frac{Zke^2}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right)$$

which an orbiting lepton interacts with.

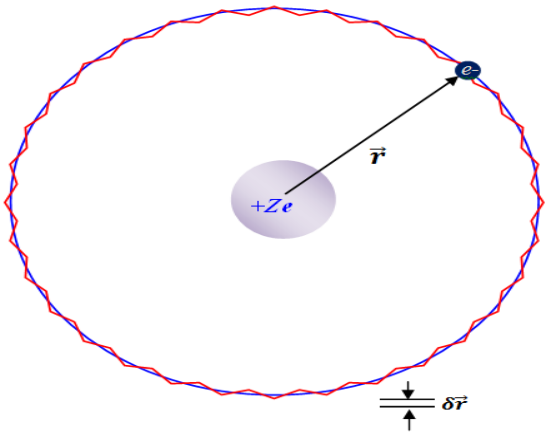

Figure 1: The oscillatory motion of an electron due to fluctuating vacuum fields (not to scale)

Thus, taking the Laplacian of the potential for the extended charge distribution, gives the following equation:

$$\vec{\nabla}^2 U(R) = \frac{Zke^2}{R^3} \quad (2)$$

The substitution of (2), equation (1) becomes:

$$U'(R) = \frac{Zke^2}{6R^3}\langle \delta r^2 \rangle_t \quad (3)$$

where $\langle \delta r^2 \rangle_t$ is mean square position of an orbiting lepton. Equation 3 is the small perturbation felt by an orbiting lepton due to its interaction with the fluctuating vacuum fields. Since the perturbation is very small as compared with the nuclear potential, its solution can be sought by applying first-order time-independent perturbation theory (Yung-Kuo, 2000; Adamu and Ngadda, 2017; Adamu and Ngadda, 2014):

$$\Delta E^{(1)}_{nlm} = \int \psi^*_{nlm} U'(R) \psi_{nlm} d\tau \quad (4)$$

where the ψ_{nlm} is the unperturbed normalized wave function and is given by

$$\psi_{nlm}(r,\theta,\phi) = \left[\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n(n+l)!}\right]^{1/2} e^{-\alpha r/2}(\alpha r)^l L^{2l+1}_{n-l-1}(\alpha r) \Upsilon^m_l(\theta,\phi) \quad (5)$$

where $\alpha = Z/na_0$, n = 1, 2, 3,… is the principal quantum number, l = 0, 1, 2, … (n – 1) is the orbital angular momentum quantum number and m = 0, ±1, ±2, … ±l is the magnetic quantum number. Regular solutions exist for $n \geq l$ +1 and the lowest state with l = 1 (called a 2p state) occurs only with n = 2. Thus, electrons can be grouped related to the quantum number n they occupy, such as 1s, 2s, 2p, 3s, 3p, 3d, and so on (Das and Sidharth, 2015; House, 2018).

Mean Square Fluctuation

The mean square fluctuation of an orbiting lepton, $\langle \delta r^2 \rangle_t$, can be obtained from the principle of quantum electrodynamics and classical physics. Classically, the motion of a simple harmonic oscillator of frequency ω is considered to give a familiar form of the simple harmonic oscillator equation of motion (Venkataram, 2013):

$$\frac{d^2 r}{dt^2} = -\omega^2 r \quad (6)$$

This corresponds to classical equation of motion for the electron displacement $(\delta\vec{r})_{\vec{k}}$ induced by a single mode of the field of wave vector $\vec{k}$ and frequency ω as:

$$m\frac{d^2}{dt^2}(\delta\vec{r})_{\vec{k}} = -e\vec{E}_{\vec{k}} \quad (7)$$

where the electric field $\vec{E}$ has components that are fluctuating with the wave vector $\vec{k}$. The deviation of an orbiting lepton from its quantum orbit is now assumed from their harmonic motion under the influence of atomic nucleus, and thus, comparing (6) and (7) yields:

$$m\omega^2\delta r_0 = e\mathcal{E}_0$$

or

$$\delta r_0 = \frac{e}{m\omega^2}\mathcal{E}_0$$

Therefore, the mean square oscillation takes the form:

$$\langle \delta r_0^2 \rangle_t = \frac{e^2}{2m^2\omega^4}\langle \mathcal{E}_0^2 \rangle_t \qquad (8)$$

According to quantum electrodynamics, a vacuum is not empty but has fluctuating electric and magnetic fields present and their zeroth energy level is quantized as a harmonic oscillator as

$$E_\omega = \frac{\hbar\omega}{2} \qquad (9)$$

where $\hbar\omega$ is the quantum of energy (Blatt, 2006; Evans and Eckardt, 2011; Dyson, 2006; Venkataram, 2013; Genet *et al.*, 201). However, the energy of fluctuating electric and magnetic fields $\mathcal{E}$ and $\mathcal{H}$ respectively, is given by:

$$E_\omega = \frac{1}{8\pi}\int(\mathcal{E}^2 + \mathcal{H}^2)d\omega = \frac{\mathcal{E}_0^2}{4\pi}\Omega \qquad (10)$$

where for a set of plane waves, $\langle \mathcal{E}^2 \rangle_t = \langle \mathcal{H}^2 \rangle_t$ (Dyson, 2006). By comparing (9) and (10), the fluctuating fields take the value:

$$\mathcal{E}_0^2 = \frac{2\pi}{\Omega}\hbar\omega \qquad (11)$$

Equation 11 represents the quantization of vacuum fields as harmonic oscillator. Therefore, using (11), the mean square oscillation (8) becomes:

$$\langle \delta r_0^2 \rangle_t = \frac{\pi}{\Omega}\frac{\hbar e^2}{m^2\omega^3} \qquad (12)$$

The mean square fluctuation is the result of non-coherent action of all components of the field:

$$\langle \delta\vec{r}^2 \rangle_t = \int \langle \delta r_0^2 \rangle_t \rho_0(\omega)d\omega \qquad (13)$$

where

$$\rho_0(\omega) = \frac{\Omega\omega^2}{2\pi^2c^3} \qquad (14)$$

is the spectral density of the zero-point radiation (Marshall, 1963; Boyer, 1975; Faria *et al.*, 2006). The full mean square fluctuation in vacuum fields is obtained by substituting (12) and (14) into (13) as follows:

$$\langle \delta\vec{r}^2 \rangle_t = \frac{\hbar e^2}{\pi m^2 c^3}\ln\left(\frac{\omega_2}{\omega_1}\right)$$

With the approximations:

$$\omega_1 = \frac{E_n}{\hbar} = \frac{(Z\alpha)^2 mc^2}{2\hbar n^2}$$

$$\omega_2 = \frac{E_T}{\hbar} = \frac{mc^2}{\hbar n^2}$$

one can evaluate the integral and have the mean square oscillation as

$$\langle \delta\vec{r}^2 \rangle_t = \frac{\hbar e^2}{\pi m^2 c^3}\ln\left[\frac{2}{(Z\alpha)^2}\right] \qquad (15)$$

where $\alpha = e^2/\hbar c$ is the fine structure constant and Z is the nuclear charge (Adamu, 2016; Adamu *et. al*., 2020). The perturbation (3) of the perturbed leptons is obtained by substituting the value of mean square oscillation $\langle \delta \vec{r}^2 \rangle_t$ of orbiting lepton (14) and presented as:

$$\lambda U'(R) = \frac{Zke^4\hbar}{6R^3\pi m^2c^3} \ln\left[\frac{2}{(Z\alpha)^2}\right] \qquad (16)$$

The shift of the energy level of finite sized nuclear due to vacuum fields' effect in the state *nlm* is calculated from the first order time-independent perturbation theory as:

$$\Delta E_{nlm}^{(1)} = \frac{Zke^4\hbar}{6R^3\pi m^2c^3} \ln\left[\frac{2}{(Z\alpha)^2}\right] \int_0^R |\psi_{nlm}|^2 r^2 dr \qquad (17)$$

where for $r \ll a_0$, $e^{-Zr/a0} \approx 1$ and the normalized wave functions are assumed to be constant over the region of integration. Thus for an extended nuclear charge distribution and the disturbances of orbiting leptons by vacuum fields fluctuations, the resulting nuclear-lepton interaction is solved using approximation methods and the small energy levels changes caused by the fluctuating vacuum fields is calculated using a computer programs (Microsoft Excel).

RESULTS AND DISCUSSION

The corrections for the states $n = 1$ (1s), $n = 2$ (2s2p) and $n = 3$ (3s3p3d), caused by the effect of vacuum fields on lepton bound to finite-size nucleus can be obtain using equation (17) and the normalized wave function (5) as follows:

$$\Delta E_{100} = \left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_1|}{1} \left(\frac{R}{a_0}\right)^0 \qquad (18a)$$

$$\Delta E_{200} = \left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_2|}{2} \left(\frac{R}{a_0}\right)^0 \qquad (18b)$$

$$\Delta E_{210} = \frac{3\pi}{2(80)}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_2|}{2} \left(\frac{R}{a_0}\right)^2 \qquad (18c)$$

$$\Delta E_{21\pm1} = \frac{3\pi}{80}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_2|}{2} \left(\frac{R}{a_0}\right)^2 (18d)$$

$$\Delta E_{300} = \frac{8}{9}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^0 \qquad (18e)$$

$$\Delta E_{310} = \frac{1}{15}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^2 \qquad (18f)$$

$$\Delta E_{31\pm1} = \frac{1}{2(15)}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^2 \qquad (18g)$$

$$\Delta E_{320} = \frac{1}{513}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^4 (18h)$$

$$\Delta E_{32\pm1} = \frac{1}{1624}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^4 \qquad (18i)$$

$$\Delta E_{32\pm2} = \frac{1}{175406}\left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \frac{|E_3|}{3} \left(\frac{R}{a_0}\right)^4 \qquad (18j)$$

where $a_0 = \hbar^2/me^2 = 5.29 \times 10^{-11}$ *m* and $a_\mu = 2.56 \times 10^{-13}$ *m* is the Bohr radius for electronic and muonic atom respectively. The simple analytical expression of calculating the effect vacuum fields on finite sized nuclei we can deduce from equations (18*a*) to (18*j*) and generalized as

$$\Delta E^e_{nlm} = C_{nlm}\gamma_z \frac{|E^e_n|}{n}\xi^l_e \quad (19)$$

$$\Delta E^\mu_{nlm} = C_{nlm}\gamma_z \frac{|E^\mu_n|}{n}\xi^l_\mu \quad (20)$$

where

$$|E^e_n| = \frac{Z^2e^2}{2n^2a_0} \quad (21)$$

and

$$\left|E^\mu_n\right| = \frac{Z^2e^2}{2n^2a_\mu} \quad (22)$$

are the ground state energy of electron and muon atoms respectively, the parameter,

$$\gamma_Z = \left(\frac{8}{9\pi}\right)(Z\alpha)^3 \ln\left[\frac{2}{(Z\alpha)^2}\right] \quad (23)$$

is the dimensionless factor and it varies with proton charge distribution. The scaling factors for electron and muon atoms are respectively,

$$\xi^l_e = \left(\frac{R}{a_0}\right)^2 \quad (24)$$

and

$$\xi^l_\mu = \left(\frac{R}{a_\mu}\right)^2 \quad (25)$$

where $R = r_0A^{1/3}$, with $r_0 = 1.2 \times 10^{-15}m$. The general expressions (19) and (20) can be applied to calculate the corrections in all the states (since the equation has terms γ_z, E_n, ξ^l and C_{nlm} that depend on *Z, n, l* and *m* respectively) of finite size of nucleus of single-electron and single-muon atoms. The values of scaling factor for single-electron and –muon finite-size nuclei are presented on Table 1, and the values of constant C_{nlm} for different quantum states *n, l* and *m,* are given in Table 2. The corrections of energy levels shift ΔE_{nlm} in 1*s*, 2*s*, 2*p*, 3*s*, 3*p* and 3*d* orbitals are calculated using equations (19) and (20) and the results are presented in Tables 3 and 4. The scaling factor ξ and the constant C_{nlm} determine the magnitude of the corrections relative to the energy levels. For large *n* and thus *l* the values of ξ grow smaller which indicates the smaller corrections (Table 1). It can be observe from Table 1 that the value of γ_Z rapidly decreases with proton number, Z. The value of C_{nlm} increases from $2p_{1/2}$ to $2p_{3/2}$ energy state and decreases from $2p_{1/2}$ to $3d_{5/2}$ states. The values of the constant C_{nlm} are also small for larger value of *n, l* and *m.* It can be observed from the Tables 3 and 4 that the corrections are more significant for *s* ($l = 0$) states. This is because the *s* states are more sensitive to the finite nuclear size effects than the *p* or *d* states. For 2*p* states, the corrections in energy states $\Delta E_{21\pm1}$ are twice of that in ΔE_{210} energy states. Similarly, the values of the corrections in ΔE_{310} sate are twice the values in $\Delta E_{31\pm1}$ states.

Table 1: The values of the scaling factor ξ_e and ξ_μ, and the dimensionless factor γ_z for various value of *Z*

Nuclides $^{A}X_Z$	ξ_μ **(10^{-4})**	ξ_e **(10^{-9})**	γ_Z **(10^{-4})**
$^{1}H_1$	0.2304	51.458	115.88
$^{6}Li_3$	0.4792	1.0704	24.762
$^{21}Na_{11}$	1.1395	2.5451	8.4017
$^{107}Ag_{47}$	3.0006	6.7017	0.0032
$^{151}Eu_{63}$	3.6479	8.1472	0.0062
$^{203}Pb_{81}$	4.3132	9.6333	0.0001
$^{235}U_{92}$	4.6954	0.1048	0.0001

Table 2: The values of the constant C_{nlm} for various energy levels ΔE_{nlm} of finite-size nucleus

ΔE_{nlm}	$\Delta E_{100}(1s_{1/2})$	$\Delta E_{200}(2s_{1/2})$	$\Delta E_{210}(1p_{1/2})$	$\Delta E_{21\pm1}(2p_{3/2})$	$\Delta E_{300}(3s_{1/2})$	$\Delta E_{310}(3p_{1/2})$	$\Delta E_{31\pm1}(3p_{3/2})$	$\Delta E_{320}(3d_{5/2})$	$\Delta E_{32\pm1}(3d_{7/2})$	$\Delta E_{32\pm2}(3d_{9/2})$
C_{nlm}	1.0000	1.0000	0.0059	0.1179	0.8888	0.0714	0.0357	0.0019	0.0006	0.0000006

Table 3: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-electron atoms

Nuclides $^{A}X_{Z}$	**The Energy Level Corrections, ΔE_{nlm} (*eV*)**									
	$\Delta E_{100}(1s_{1/2})$	$\Delta E_{200}(2s_{1/2})$	$\Delta E_{210}(1p_{1/2})$	$\Delta E_{21\pm1}(2p_{3/2})$	$\Delta E_{300}(3s_{1/2})$	$\Delta E_{310}(3p_{1/2})$	$\Delta E_{31\pm1}(3p_{3/2})$	$\Delta E_{320}(3d_{5/2})$	$\Delta E_{32\pm1}(3d_{7/2})$	$\Delta E_{32\pm2}(3d_{9/2})$
$^{1}H_{1}$	1.57×10^{-5}	7.88×10^{-6}	2.46×10^{-26}	4.92×10^{-25}	4.66×10^{-6}	9.93×10^{-26}	4.96×10^{-26}	2.64×10^{-27}	8.34×10^{-28}	8.34×10^{-31}
$^{6}Li_{3}$	3.03×10^{-3}	1.51×10^{-3}	2.04×10^{-23}	4.09×10^{-22}	8.98×10^{-4}	8.26×10^{-23}	4.13×10^{-23}	2.19×10^{-24}	6.94×10^{-25}	6.94×10^{-28}
$^{21}Na_{11}$	1.38×10^{0}	6.91×10^{-1}	5.28×10^{-20}	1.05×10^{-18}	4.09×10^{-1}	2.13×10^{-19}	1.06×10^{-19}	5.67×10^{-21}	1.79×10^{-21}	1.79×10^{-24}
$^{107}Ag_{47}$	9.72×10^{2}	4.86×10^{2}	2.57×10^{-16}	5.14×10^{-15}	2.88×10^{2}	1.03×10^{-15}	5.19×10^{-16}	2.76×10^{-17}	8.73×10^{-18}	8.73×10^{-21}
$^{151}Eu_{63}$	3.33×10^{3}	1.66×10^{3}	1.30×10^{-15}	2.61×10^{-14}	9.88×10^{2}	5.27×10^{-15}	2.63×10^{-15}	1.40×10^{-16}	4.42×10^{-17}	4.42×10^{-20}
$^{203}Pb_{81}$	9.10×10^{3}	4.55×10^{3}	4.98×10^{-15}	9.95×10^{-14}	2.69×10^{3}	2.01×10^{-14}	1.00×10^{-14}	5.34×10^{-16}	1.68×10^{-16}	1.68×10^{-19}
$^{235}U_{92}$	1.46×10^{4}	7.34×10^{3}	9.53×10^{-15}	1.90×10^{-13}	4.35×10^{3}	3.84×10^{-14}	1.92×10^{-14}	1.02×10^{-15}	3.23×10^{-16}	3.23×10^{-19}

Table 4: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-muon atoms

Nuclides $^{A}X_{Z}$	**The Energy Level Corrections, ΔE_{nlm} (*eV*)**									
	$\Delta E_{100}(1s_{1/2})$	$\Delta E_{200}(2s_{1/2})$	$\Delta E_{210}(1p_{1/2})$	$\Delta E_{21\pm1}(2p_{3/2})$	$\Delta E_{300}(3s_{1/2})$	$\Delta E_{310}(3p_{1/2})$	$\Delta E_{31\pm1}(3p_{3/2})$	$\Delta E_{320}(3d_{5/2})$	$\Delta E_{32\pm1}(3d_{7/2})$	$\Delta E_{32\pm2}(3d_{9/2})$
$^{1}H_{1}$	2.21×10^{-2}	1.11×10^{-2}	4.98×10^{-12}	9.95×10^{-11}	6.57×10^{-3}	7.18×10^{-3}	3.59×10^{-3}	1.91×10^{-4}	6.03×10^{-5}	6.03×10^{-8}
$^{6}Li_{3}$	3.83×10^{1}	1.91×10^{1}	1.65×10^{-6}	3.31×10^{-5}	1.13×10^{1}	1.11×10^{2}	5.59×10^{1}	2.97×10^{0}	9.39×10^{-1}	9.39×10^{-4}
$^{21}Na_{11}$	2.35×10^{6}	1.17×10^{5}	4.63×10^{0}	9.27×10^{1}	6.97×10^{5}	9.22×10^{6}	4.61×10^{7}	2.45×10^{5}	7.75×10^{4}	7.75×10^{1}
$^{107}Ag_{47}$	3.02×10^{9}	1.51×10^{9}	4.18×10^{7}	8.36×10^{9}	8.96×10^{8}	2.16×10^{12}	1.08×10^{12}	5.75×10^{10}	181×10^{10}	1.81×10^{7}
$^{151}Eu_{63}$	1.86×10^{10}	9.32×10^{9}	8.86×10^{8}	177×10^{10}	5.52×10^{9}	2.39×10^{13}	1.23×10^{13}	6.37×10^{11}	2.01×10^{11}	2.01×10^{8}
$^{203}Pb_{81}$	8.41×10^{10}	4.26×10^{10}	1.08×10^{10}	2.17×10^{11}	2.49×10^{10}	1.78×10^{14}	8.92×10^{13}	4.75×10^{12}	1.49×10^{12}	1.49×10^{9}
$^{235}U_{92}$	1.75×10^{11}	8.75×10^{10}	3.66×10^{10}	7.32×10^{11}	5.19×10^{10}	4.79×10^{14}	2.42×10^{14}	1.28×10^{13}	4.03×10^{12}	402×10^{9}

Table 5: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-electron atoms

Nuclei $^{A}X_{Z}$	**The Logarithmic Value of the Energy States ΔE_{nlm} of Electron (*eV*)**									
	$\Delta E_{100}(1s_{1/2})$	$\Delta E_{200}(2s_{1/2})$	$\Delta E_{210}(1p_{1/2})$	$\Delta E_{21\pm1}(2p_{3/2})$	$\Delta E_{300}(3s_{1/2})$	$\Delta E_{310}(3p_{1/2})$	$\Delta E_{31\pm1}(3p_{3/2})$	$\Delta E_{320}(3d_{5/2})$	$\Delta E_{32\pm1}(3d_{7/2})$	$\Delta E_{32\pm2}(3d_{9/2})$
$^{1}H_{1}$	-4.8024	-5.1035	-25.6087	-24.3080	-5.3307	-25.0029	-25.3040	-26.5779	-27.0785	-30.0785
$^{6}Li_{3}$	-2.5184	-2.8195	-22.6885	-21.3878	-3.0467	-22.0828	-22.3838	-23.6577	-24.1583	-27.1583
$^{21}Na_{11}$	0.1407	-0.1603	-19.2770	-17.9764	-0.3876	-18.6713	-18.9723	-20.2463	-20.7469	-23.7469
$^{107}Ag_{47}$	2.9877	2.6867	-15.5891	-14.2884	2.4594	-14.9833	-15.2844	-16.5583	-17.0589	-20.0589
$^{151}Eu_{63}$	3.5233	3.2223	-14.8838	-13.5832	2.9950	-14.2781	-14.5791	-15.8531	-16.3537	-19.3537
$^{203}Pb_{81}$	3.9590	3.6580	-14.3026	-13.0019	3.4308	-13.6968	-13.9979	-15.2718	-15.7724	-18.7724
$^{235}U_{92}$	4.1670	3.8660	-14.0209	-12.7202	3.6387	-13.4151	-13.7162	-14.9901	-15.4907	-18.4907

Table 6: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-muon atoms

Nuclei $^{A}X_{Z}$	**The Logarithmic Value of the Energy States ΔE_{nlm} of Muon (*eV*)**									
	$\Delta E_{100}(1s_{1/2})$	$\Delta E_{200}(2s_{1/2})$	$\Delta E_{210}(1p_{1/2})$	$\Delta E_{21\pm1}(2p_{3/2})$	$\Delta E_{300}(3s_{1/2})$	$\Delta E_{310}(3p_{1/2})$	$\Delta E_{31\pm1}(3p_{3/2})$	$\Delta E_{320}(3d_{5/2})$	$\Delta E_{32\pm1}(3d_{7/2})$	$\Delta E_{32\pm2}(3d_{9/2})$
$^{1}H_{1}$	-1.6539	-1.9550	-11.3025	-10.0019	-2.1822	-2.1438	-2.4449	-3.7188	-4.2194	-7.2194
$^{6}Li_{3}$	1.5843	1.2833	-5.7803	-4.4796	1.0560	2.0487	1.7476	0.4737	-0.0269	-3.0269
$^{21}Na_{11}$	5.3720	5.0709	0.6665	1.9672	4.8437	6.9649	6.6638	5.3899	4.8893	1.8893
$^{107}Ag_{47}$	9.4804	9.1794	7.6220	8.9226	8.9521	12.3347	12.0337	10.7598	10.2592	7.2592
$^{151}Eu_{63}$	10.2705	9.9694	8.9476	10.2483	9.7422	13.3793	13.0782	11.8043	11.3037	8.3037
$^{203}Pb_{81}$	10.9245	10.6235	10.0374	11.3380	10.3962	14.2516	13.9505	12.6766	12.1760	9.1760
$^{235}U_{92}$	11.2431	10.9420	10.5639	11.8646	10.7148	14.6808	14.3797	13.1058	12.6052	9.6052

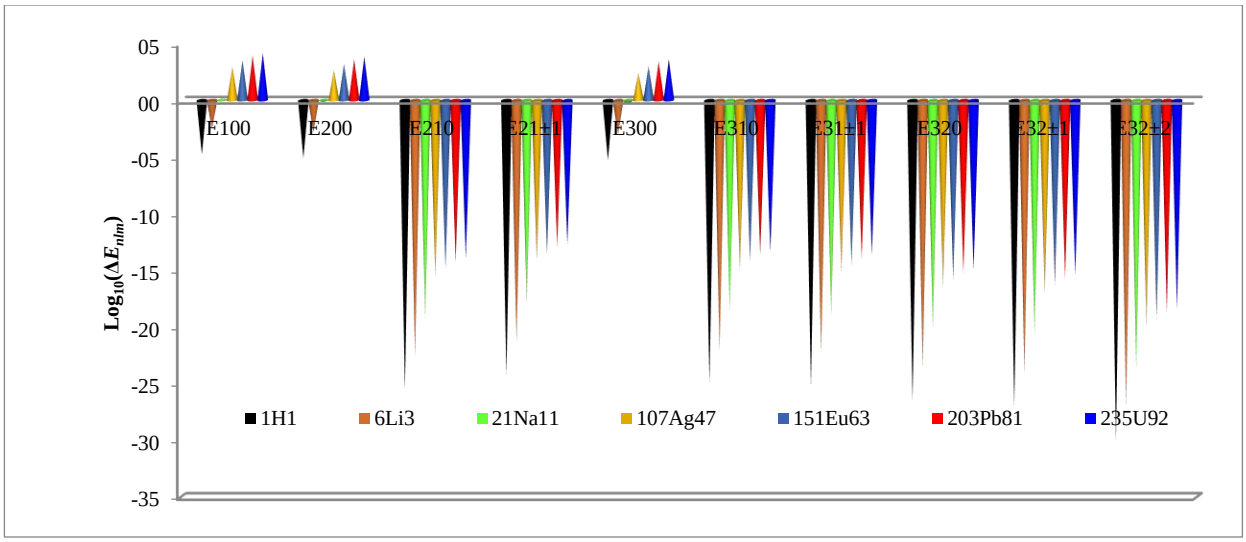

Figure 2: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-electron hydrogen-like atoms

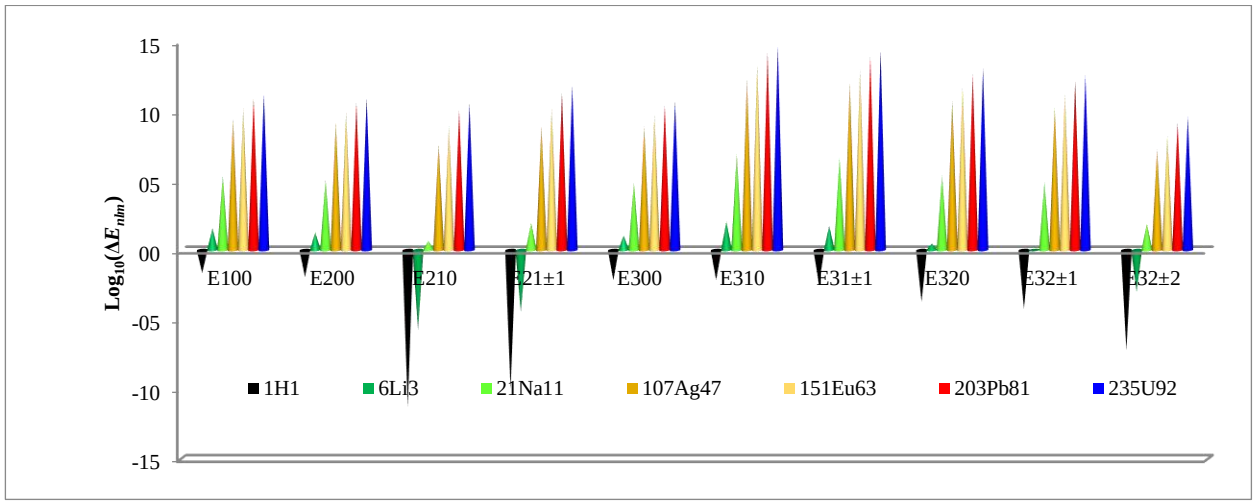

Figure 3: The corrections to energies of 1*s* 2*s*2*p* 3*s*3*p*3*d* states of single-muon hydrogen-like atoms

An observation in Figures 2 and 3 showed that the effects of vacuum electromagnetic fields on finite-size nuclei are more significant in the case of higher Z and muonic atoms. This comes as a result of the more overlap of muon states with finite-size nuclei than that of electron states. The nuclear finite-size corrections can be more important for higher atomic nuclei and muonic atoms, since for muon, $m\mu$ = 207 *me*. A direct effect of the vacuum field fluctuations and its experimental verification constitutes strong support of quantum electrodynamics.

The details of atomic spectra need many additional corrections including those due to quantum electrodynamics effects and nuclear structure effects (El Shabshiry *et al.*, 2015). These corrections have to be investigated by several authors (see for example (Welton, 1948; Weisskopf, 1949; Das and Sidharth, 2015) through the use of perturbation theory as an approximation method. The Quantum mechanics calculation of vacuum field fluctuations due to Bethe, (1947) and Welton, (1948) results in energy shifts:

$$\Delta E_{nlm} = |E_n| \frac{8}{3\pi} \alpha^3 \frac{Z^2}{n} \ln\left[\frac{V_{max}}{V_{min}}\right]$$

The energy level shift due to vacuum field's fluctuation was quantitatively formulated by Welton (1948) as:

$$\Delta E_{nlm} = \frac{8}{3\pi}\frac{Z^4\alpha^5}{n^3} ln\left(\frac{1}{Z\alpha^2}\right)\frac{m}{2}\delta_{l,0}$$

This energy shift corresponds to the frequency of $f \approx 1000$ *MHz* for hydrogen atom (Z = 1) when an electron is in $n = 2$ and $l = 0$ energy states (Das and Sidharth, 2015).

By virtue of the modified Dirac equation, Das and Sidharth (2015) obtained the energy shift as

$$\Delta E_n = |E_n|\frac{8}{3\pi}\alpha^3\frac{Z^2}{n}\log\left(\frac{fmc^2}{h\nu_0}\right)$$

and

$$\Delta E_n = |E_n|\frac{8}{3\pi}\alpha^3\frac{Z^2}{n}\ln\left[\frac{f}{(Z\alpha)^2}\right]$$

corresponds to the frequency $f \approx 1056$ *MHz* which is very nearly equal to the *Lamb shift*. From different perspectives, Peterman and Karshenboim, (1994) obtained such frequency $f = 1057.911 \pm 0.011$ *MHz* and 1057.8576(21) *MHz* respectively. Weisskopf, (1949) obtained the direct effect of vacuum field fluctuations as:

The energy shift δE_n due to fluctuating vacuum fields was determined by Pardy, (2016) as:

$$\delta E_n \approx \frac{2}{3\pi^2}\frac{\alpha^3}{n^2}\ln\left(\frac{\omega_2}{\omega_1}\right)$$

These values are so close to an outstanding triumph of both experimental and theoretical physics.

CONCLUSION

From the results obtained, we conclude that the newly proposed formula would be suitable for calculating the energy level corrections in both muonic and electronic atoms caused by the fluctuating vacuum fields and that the vacuum field fluctuations have more effect on higher Z and in muonic hydrogen-like atoms. The obtained energy level corrections calculated from the derived formula are an additional energy level correction that has to be investigated for reanalyzing the existing measured nuclear structure effects and for comparison with future spectroscopic investigations.

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