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Research Article

Proposes a hybrid cosine-exponentiated Student's t innovation for GARCH models and tests it on Nigerian stock-exchange data

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ABSTRACT

This study introduces the Cosine Exponentiated Generalized Student's t (CEGST) distribution as a novel error innovation for GARCH-type models to better capture financial market volatility. It evaluates GARCH(1,1), GARCH(1,2), and GARCH(2,1) models using simulated data and daily returns from the Nigerian Stock Exchange between 7 February 2012 and 28 November 2023 making a total of 2,948 observations. Models were estimated in R using the 'fgarch' package and the performance was assessed via AIC, BIC, MAE, MSE and RMSE. Simulation results show the GARCH(1,2) CEGST model (Case 1) achieved the best performance with lowest values across all criteria: AIC (–1360.182), BIC (–1389.629), MAE (0.09461), MSE (0.01652), RMSE (0.12853), indicating improved fit through an additional GARCH lag. In contrast, empirical results on NSE data favored the GARCH(2,1) CEGST model, achieving superior in-sample performance with AIC (–46,717.62), BIC (–46,663.72), MAE (0.00636), MSE (9.31×10^{-5}) and RMSE (0.00965). Residual diagnostics revealed no serial correlation or ARCH effects at the 5% level. The CEGST-based GARCH models effectively capture key features of volatility in emerging markets such as cyclical behavior, heavy tails and market shocks. The findings demonstrate the efficacy of the CEGST distribution in portraying the complex volatility patterns typical of developing financial markets.

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INTRODUCTION

The Nigerian Exchange Group (NGX), formerly the Nigerian Stock Exchange, plays a central role in capital mobilization and economic growth in Nigeria. Despite regulatory reforms aimed at enhancing transparency and expanding investor participation, the financial market remains highly volatile (Garba, 2924). This volatility is largely driven by macroeconomic instability, political risk, limited market liquidity, and low investor education (CBN, 2023; NSE Market Report, 2022). As a result, accurate volatility modeling and forecasting are essential for portfolio optimization, derivative pricing and effective risk management.

Volatility modeling has long been a cornerstone of financial econometrics, spurred by stylized facts such as volatility clustering, fat tails (leptokurtosis) and asymmetry (Cont, 2001). Bollerslev (1986) Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model remains foundational in capturing these features. However, traditional GARCH models often rely on Gaussian or symmetric Student's t innovations, which may be overly restrictive for emerging markets like Nigeria (Brooks, 2014).

To overcome these limitations, researchers have explored GARCH-type models incorporating heavy-tailed and skewed error innovations. Yet relatively few have introduced time-varying or cyclical patterns in the innovation distribution. This study addresses that gap by proposing the Cosine Exponentiated Generalized Student's t (CEGST) distribution, a dynamic error innovation that uses cosine modulation to model time-dependent volatility cycles, structural breaks, and seasonal effects prevalent in financial data (Adenomon et al., 2022). CEGST-based GARCH models allow for more realistic tail risk simulation and long-term volatility persistence, both common in Nigerian financial markets.

Although extensive work has advanced the GARCH model family including innovations addressing asymmetry, long memory and regime shifts the specification of the error distribution remains underexplored. The assumption of normally distributed errors, though computationally simple, inadequately captures the excess kurtosis, skewness and tail risks often observed in real-world returns. Researchers like Dikko and Agboola (2017) and Wang and Chen (2022) have emphasized the need for more flexible error structures, while Stavrianos (2024) argued for enhancements to support long-term forecasting.

Practical constraints in traditional GARCH models have led scholars such as Nguyen and Park (2023) and Smith and Doe (2023) to critique their limited adaptability, especially in emerging markets. Further efforts by Chen *et al*. (2019) and Chung (2024) highlight the use of machine learning and non-standard error distributions such as skewed and leptokurtic innovations to improve predictive accuracy. Despite these advances, functional transformations like sine or cosine exponentiation are rarely employed in innovation modeling, even though they offer valuable tools for representing cyclical volatility. To address this gap, this study introduces the CEGST distribution, combining heavy tails and periodic modulation to deliver a richer and more realistic portrayal of volatility behavior.

This study presents a novel approach to modeling financial market volatility by introducing the CEGST distribution, which captures recurring patterns and heavy-tailed behaviours often observed in Nigeria's stock market. The research evaluates predictive performance using statistical diagnostics and validation tests by analyzing three variants of the GARCH model; (1,1), (1,2) and (2,1). Unlike most existing studies that focus on developed markets, this paper addresses the volatility dynamics of an emerging economy, where market fluctuations are intensified by structural frictions. Prior research on Nigerian data, such as Atoi (2014), recognized the limitations of Gaussian assumptions and highlighted the potential of Student's t innovations, though without considering periodic enhancements. This work bridges that gap by applying the hybrid CEGST structure to Nigerian Stock Exchange returns, offering both methodological innovation and practical insights for financial analysts and researchers.

MATERIALS AND METHODS

This research explores the modeling and forecasting of volatility in the Nigerian Stock Exchange (NSE) through sophisticated

GARCH-type models. Data employed consists of closing values of the Nigerian Stock Exchange All Share Index (NSE-ASI) spanning 7th February 2012 to 28th November 2023. The reason for choosing the NSE-ASI is its comprehensive coverage of all listed shares, which numbered 169 companies at the time of this research. Prices were adjusted to continuously compounded returns via log-differencing to ensure stationarity.

The Nigerian Stock Exchange, established in 1960 as the Lagos Stock Exchange and later renamed in 1977, is currently the second-largest on the African continent in terms of market capitalization. The market is governed by the Securities and Exchange Commission (SEC), and it is brought about through monitoring mechanisms that are capable of discovering manipulative trades and price distortions. This research utilizes GARCH(1,1) GARCH (1,2) and GARCH (2,1) models with cosine exponentiated generalized transformed Student's t error innovation in modeling Nigerian Stock Exchange return volatility on the basis of log-differenced daily prices. Parameters of the model are maximally estimated and performance is tested with AIC, BIC, MSE, MAE, and RMSE to compare it with standard GARCH models.

Generalized Autoregressive Conditional Heteroscedastic Model (GARCH)

Bollerslev (1986) and Taylor (1986), the Model If an autoregressive moving average model (ARMA model) is found for the error variance (σ_t^2), the model is a generalized autoregressive conditional heteroskedasticity (GARCH) model. The general specification of GARCH (p, q) is as follows

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + ... \ \alpha_q\varepsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + ... \ \beta_p\sigma_{t-p}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-1}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-1}^2 \tag{1}$$

where α_i = coefficients of the ARCH component; and β_i = coefficient of the GARCH component. The three parameters (α_0, α_i and β_i) are restricted to be positive (assumed to be non-negative) and $\alpha_i + \beta_i < 1$ to achieve stationarity The test for the present of GARCH effect is established in two steps: First, we estimate the best fitting AR (q) model which is the AR (q) model with a lag order that gives the lowest Aikaike and Swartz information and highest log-likelihood ratio.

Cosine Exponentiated Generalized Students T Error Innovation Distribution

The Cosine-Exponentiated Generalized Student's t (CEGSt) distribution is a novel, computationally efficient, and flexible competitor for financial return innovation modeling under the GARCH setting. Its pioneering design allowing for tail thickness, skewness, and periodicity ensures optimal empirical fit to financial data. The flexibility of the distribution reveals itself through improved volatility forecasting, improved model fitting, and improved risk measurement. The GARCH-CEGSt model is an excellent addition to the toolkit of modern financial econometrics as it is.

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \gamma_i\cos(\delta_t)^\lambda \tag{2}$$

Where σ_t^2 is the conditional variance, ω is the baseline constant variance. α capture the impact of past squared shocks on volatility., β capture the persistent of past volatility, $\gamma\cos(\delta_t)^\lambda$ introduced a periodic component to capture cyclical fluctuation in volatility.

Validity of Cosine Exponentiated Generalized Error Innovation Distribution

$$\int_{-\infty}^{\infty}\frac{\pi}{2}\cdot\frac{\lambda\theta}{(2+x^2)^{3/2}}\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta-1}\cdot\sin\left\{\frac{\pi}{2}\left[1-\left(1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right)^{\lambda}\right]^{\theta}\right\}dx \quad (3)$$

$$k=\frac{\pi}{2}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta}$$

as $x\to-\infty, \qquad \text{p}=0$

as $x\to\infty, \qquad \text{p}=\frac{\pi}{2}$

$$\frac{dk}{dx}=\frac{\pi}{2}\cdot\frac{\lambda\theta}{(2+x^2)^{3/2}}\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta-1} \quad (4)$$

$$dx=\frac{dk}{\frac{\pi}{2}\cdot\frac{\lambda\theta}{(2+x^2)^{3/2}}\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta-1}}$$

$$\int_{-\infty}^{\infty}\frac{\pi}{2}\cdot\frac{\lambda\theta}{(2+x^2)^{3/2}}\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta-1}\cdot$$

$$\times\sin(k)\cdot\frac{dk}{\frac{\pi}{2}\cdot\frac{\lambda\theta}{(2+x^2)^{3/2}}\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{x}{\sqrt{2+x^2}}\right)\right]^{\lambda}\right\}^{\theta-1}}$$

The integral will simplify to

$$\int_{0}^{\frac{\pi}{2}}\sin(k)\,dk=\left[-\cos(k)\right]_{0}^{\frac{\pi}{2}}=-\cos\left(\frac{\pi}{2}\right)+\cos(0)=-0+1=1$$

Therefore, the PDF of the proposed Cosine Exponentiated Generalized Error Innovation Distribution is valid

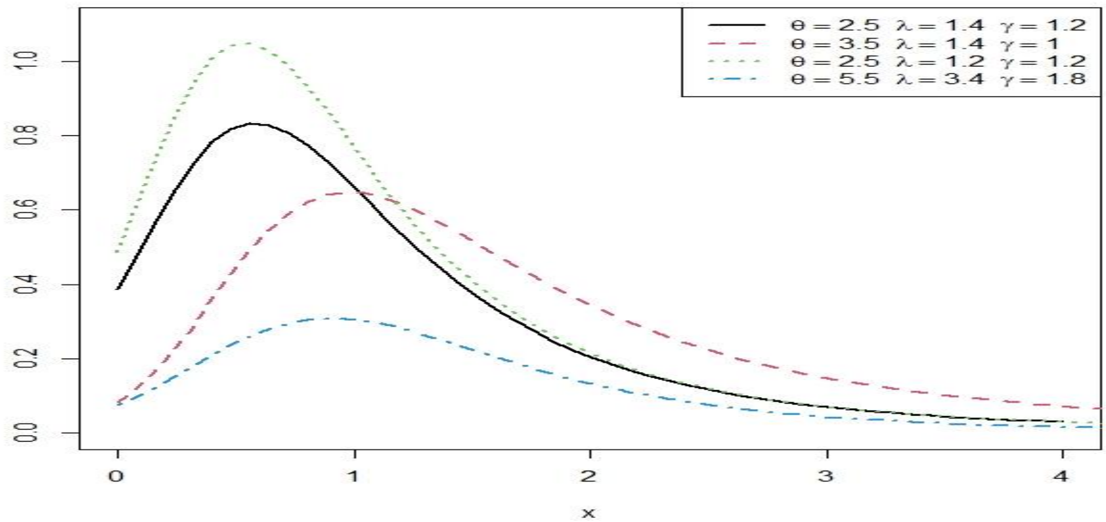

Figure1: Plot of shape parameter Cosine Exponentiated Generalized Student's t error Innovation distribution

Figure 1 reveals the shape dynamics of the Cosine Exponentiated Generalized Student's t (CEGST) error innovation distribution for varying parameter settings. The figure represents the way the shape of the distribution changes with adjustments in three control parameters: λ is the exponentiation parameter, which controls the flattening and spreading of the peak of the distribution, γ is the tail-weight parameter, which regulates the heaviness of the tails and the peakedness of the distribution, and θ is the cosine-shift parameter, which determines the periodicity and shifts the position and spread of the distribution.

The green dotted line (with comparatively small values of λ and θ) in the figure shows a very sharp peak and heavier tails, reflecting greater concentration around the center. On the other hand, the blue dash-dotted graph (for high values of λ, γ, and θ) appears flatter, indicating a more spread out and lighter-tailed shape. Such stability highlights the flexibility of the CEGST distribution to explain a wide range of empirical behaviors from highly peaked to effectively uniform error structures making it sufficient to summarize complex, time-changing volatility patterns in financial return series.

Evaluation, Comparison of Models and Selection Criteria

At each scenario of specification, underline distributions and sample size, the models will be examined on the simulated data and compared using the finite sampling properties of models which are mean absolute error (MAE) and mean squared error (MSE), Root mean square error (RMSE), Akaike Information criteria (AIC) and Bayesian Information criteria (BIC). The estimator with minimum criteria under different scenario of simulations will be taken as the best.

Mean Square Error (MSE)

$$MSE = \sum_{i=1}^{n} \frac{(y_i - \hat{y})}{n} \tag{5}$$

where y_i is the actual (true) value, $\hat{y}$ is the predicted value and N is the total number of observations.

Evaluation of Volatility Forecasts

Evaluating the performance of different forecasting models plays a very important role in choosing the most accurate models. The most widely used evaluation measures is Root Mean Square Error (MSE) given as:

$$RMSE = \sqrt{\frac{\sum_{t=T+1}^{T+n}(\hat{\sigma}_t^2 - \sigma_t^2)^2}{n}} \quad (6)$$

Where, n is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$ and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively

Mean Absolute Error

$$MAE = \frac{\sum_{i=1}^{n}|\varepsilon_i|}{n} \quad (7)$$

where ε_i is the difference between predicted value and actual value, and n is the total number of observations.

Akaike Information (AIC)

There are several information criteria available to determine the best model of autoregressive process. All of them are likelihood based; the well-known is Akaike information criterion (AIC). Akaike Information Criterion is a measure of the relative quality of a statistical model for a given set of data. That is, given a collection of models for the data, AIC estimates the quality of a model relative to each of the other models. Hence, AIC provides a means for model selection. Supposing we have a statistical model of some data, let k be the number of estimated parameters included in the model and n, sample size, then, the AIC value of the model is

$AIC = n\ln(\hat{\sigma}^2) + 2k$ (See Akaike, 1973 cited by Burnham and Anderson, 2001 and Tsay, 2010) where

$$\hat{\sigma}^2 = \frac{\text{Residual Sum of Squares}}{n}.$$

Bayesian Information Criterion (BIC)

The $\ln(\text{likelihood})$ of the model given in the data, is readily available in statistical output, and reflects the overall fit of the model. The BIC is a statistical measure that is used in model selection and hypothesis testing to detect the best model among all other model. This method strikes a balance between model fit and complexity and aims to identify the most economical and informative model

BIC= $k\log(n) - 2\log(L(\theta))$ (See Ba 1979)

n = the sample size,k = the number of parameters that the model estimates

$L(\theta)$ = the maximized value of the likelihood of the model

RESULT AND DICUSSION

Simulation Results and Model Performance Evaluation

Table 1 reveals that the simulation findings for the GARCH(1,1), GARCH(1,2) and GARCH(2,1) models under the Cosine Exponentiated Generalized Student's t (CEGST) error innovation distribution. There were six parameter configurations for each model (Cases 1–6) for sample sizes $n = 200, 400, 600, 800, 1000$. AIC, BIC, MAE, MSE and RMSE were used to evaluate the performance of the model.

Table 1: GARCH model with Cosine Exponentiated Generalized Student's t Error Distribution

MODEL	CASE	PARAMETERS $(\omega,\alpha,\beta,\gamma,\delta\ \lambda)$ (n= 200,400,600.800,1000)	AIC	BIC	MAE	MSE	RMSE
GARCH (1,1)	Case 1	(0.01,0.05,0.05,0.05,0.05,0.05)	-1206.26	-1230.80	0.1017	0.0190	0.1412
	Case 2	(0.05,0.04,0.05,0.05,0.05,0.05)	-325.052	-300.513	0.2201	0.0857	0.2927
	Case 3	(0.05,0.05,0.01,0.03,0.04,0.05)	-428.218	-403.679	0.2322	0.1035	0.3217
	Case 4	(0.05,0.05,0.05,0.05,0.05,0.05)	-454.757	-430.218	0.2330	0.1048	0.3237
	Case 5	(0.05,0.05,0.05,0.05,0.04,0.05)	-394.224	-369.686	0.2279	0.1028	0.3206
	Case 6	(0.05,0.05,0.05,0.05,0.05,0.02)	-437.585	-413.046	0.2351	0.0962	0.3102
GARCH (1,2)	Case 1	(0.01,0.05,0.05,0.05,0.05,0.05)	-1360.18	-1389.63	0.0946	0.0165	0.1285
	Case 2	(0.05,0.05,0.05,0.05,0.05,0.05)	-450.146	-420.699	0.2346	0.1050	0.3240
	Case 3	(0.05,0.05,0.03,0.05,0.05,0.05)	-506.811	-477.365	0.2416	0.1027	0.3204
	Case 4	0.05,0.05,0.05,0.02,0.05,0.05)	-399.697	-370.251	0.2295	0.0947	0.3077
	Case 5	(0.05,0.05,0.05,0.05,0.05,0.05)	-458.080	-428.634	0.2355	0.1033	0.3214
	Case 6	(0.05,0.05,0.05,0.05,0.05,0.05)	-402.897	-373.451	0.2280	0.1001	0.3164
GARCH (2,1)	Case 1	(0.01,0.05,0.05,0.05,0.05,0.05)	-1205.73	-1235.17	0.1022	0.0216	0.1469
	Case 2	(0.05,0.02,0.05,0.05,0.05,0.05)	-442.123	-412.677	0.2321	0.1012	0.3182
	Case 3	(0.05,0.05,0.03,0.05,0.05,0.05)	-447.569	-418.123	0.2332	0.1038	0.3222
	Case 4	(0.05,0.05,0.05,0.03,0.05,0.05)	-438.949	-409.503	0.2334	0.0979	0.3129
	Case 5	(0.05,0.05,0.05,0.05,0.04,0.05)	-481.779	-452.332	0.2384	0.1033	0.3214
	Case 6	(0.05,0.05,0.05,0.05,0.05,0.02)	-499.788	-470.341	0.2395	0.1064	0.3263

Table 1 shows that the GARCH(1,2) model performed better than two other models competitors in all the evaluation measures, with Case 1 being the best specification (AIC = -1360.182, BIC = -1389.629, RMSE = 0.1285).Therefore the GARCH(1,1) model Case 1 also performed well with (RMSE = 0.1412), it was surpassed by GARCH(1,2). The GARCH(2,1) model was relatively weaker, with its best RMSE (0.3129 in Case 4) significantly higher. These findings shows that the GARCH(1,2) model Case 1 with CEGST innovations as the best fit for volatility modeling and forecasting for this situation

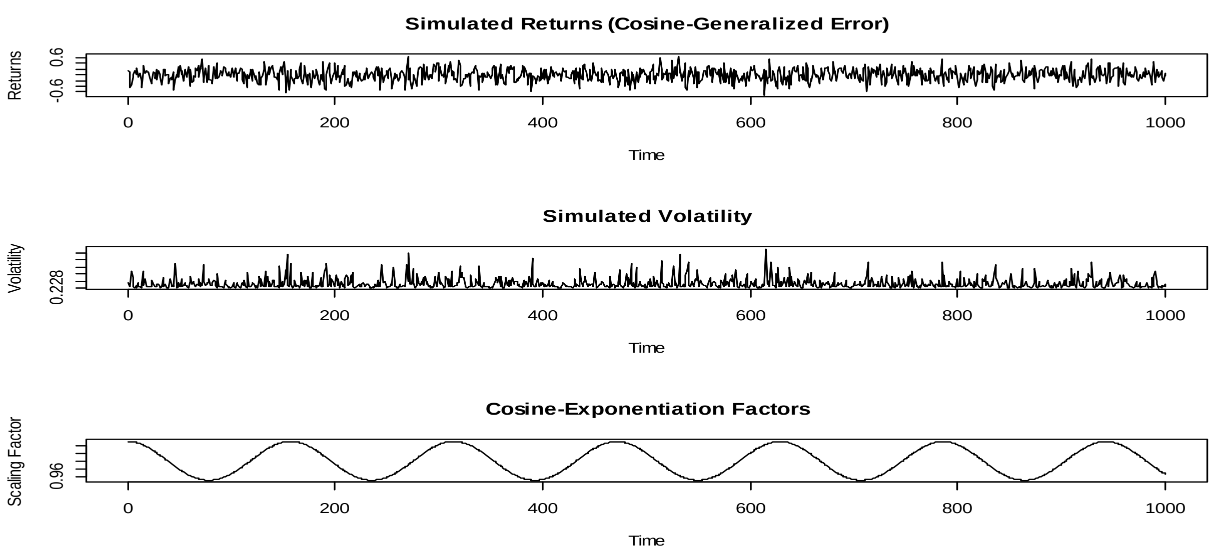

Figure 2: Plot of Cosine Exponentiated Generalized Student's t Error Innovation WITH GARCH (1,1) Model

Figure 2 reveal that the simulated traces from a GARCH(1,1) with cosine-exponentiated generalized Student's t error innovations. The top panel illustrates the series of returns, with heavy tails and sporadic big shocks indicating the capriciousness of financial returns. The middle panel illustrates conditional volatility estimates, with clear volatility clustering alternate phases of serenity and chaos indicating true market dynamics. The bottom panel traces the time-varying, cosine-modulated shape parameter, which periodically switches the tail behavior of the error distribution and therefore introduces periodic dynamics akin to seasonality in volatility impacts. The coupling of the heavy-tailed t-distribution with the cosine exponentiation adds power to the model to cover extreme events and skewness, thereby improving risk forecasting tools such as Value-at-Risk (VaR). Overall, the figure shows that the proposed model correctly captures the key stylized facts of financial returns: fat tails, volatility clustering, and cyclical volatility, and offers a more flexible and realistic alternative to standard GARCH models.

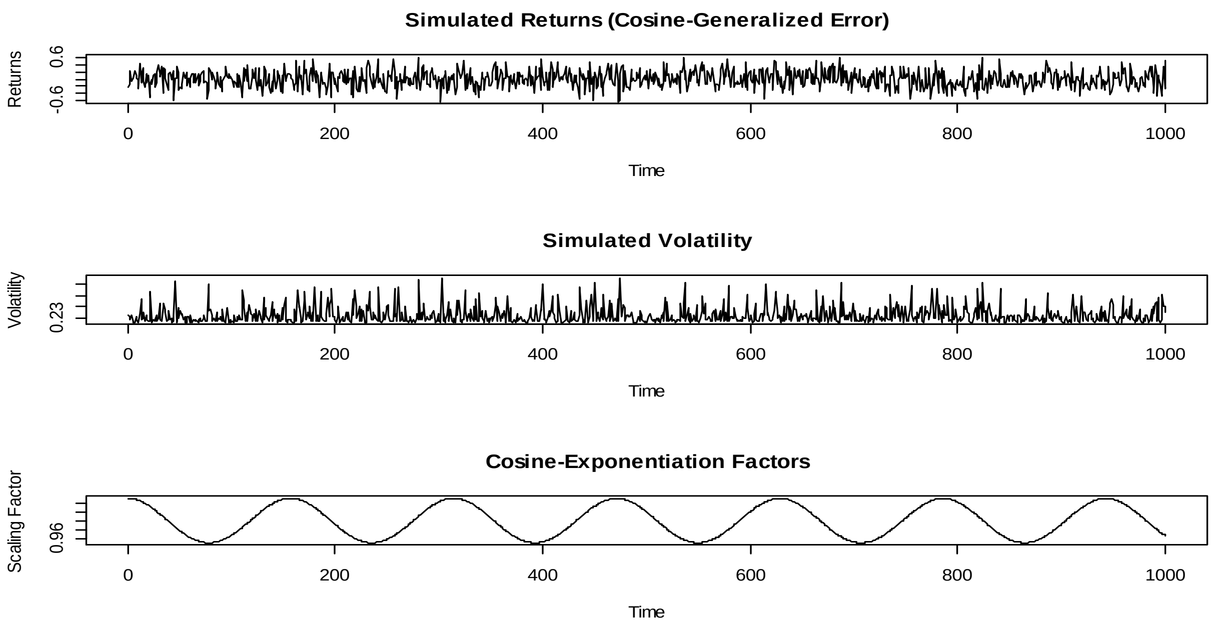

Figure 3: Plot of Cosine Exponentiated Generalized Student's t Error Innovation WITH GARCH (1,2) Model

Figure 3 the GARCH(1,2) model simulated return series using cosine-generalized exponentiated Student's t error innovations. The top panel shows the return series with heavy tails, mean reversion, and occasional extreme spikes typical of financial return dynamics. The middle panel depicts the conditional volatility time-varying term (σ_t) with the two lag GARCH terms and single ARCH term enabling the model to more effectively model longer volatility persistence and clustering than a GARCH(1,1). The bottom panel depicts the dynamic shape parameter with cosine modulating time-varying skewness and kurtosis in the error distribution. This modulation changes the tail behavior and asymmetry of the distribution cyclically following regimes of markets or patterns of risk seasonality. Overall, the figure notes that the combination of a GARCH(1,2) structure with a cosine-generalized exponentiated t innovation improves the model's ability to capture real-world phenomena such as volatility clustering, fat tails, and cyclical market risk, lowering misspecification errors of risk measures

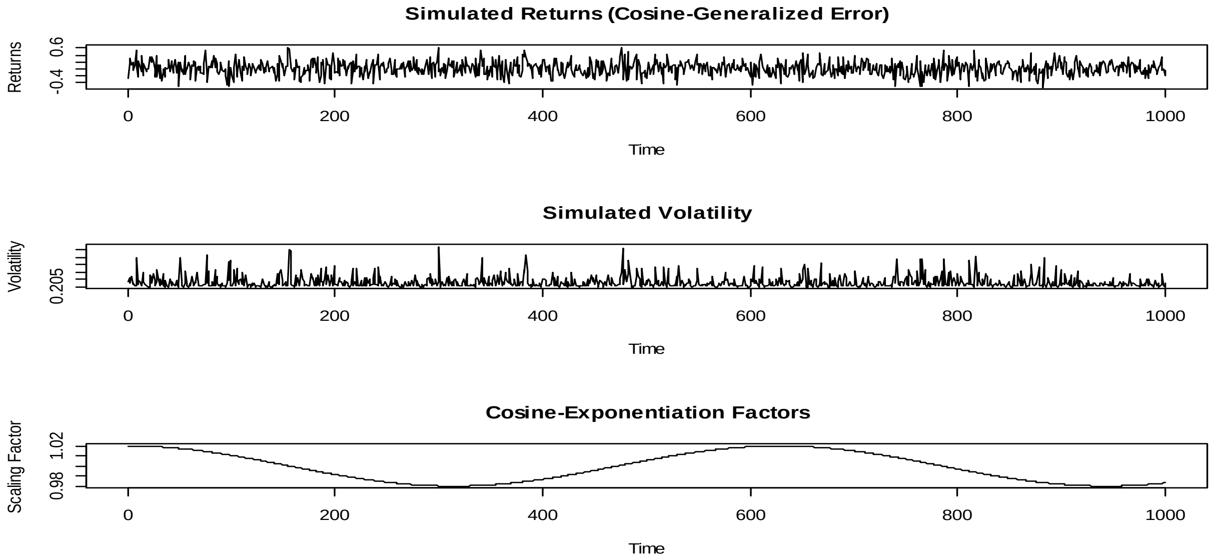

Figure 4: Plot of Cosine Exponentiated Generalized Student's t Error Innovation WITH GARCH (2,1) Model

Figure 4 graphs the GARCH(2,1) model's simulated series of returns with cosine-generalized exponentiated Student's t error innovations. The top panel graphs return behavior that is indicative of sudden breaks from the mean, which captures the heavy-tailed and mean-reverting property typical of financial returns. The middle plot represents the conditional volatility over time (σ_t), where two GARCH terms and one ARCH term allow the model to reflect long memory and long volatility regimes following shocks in the market. The bottom plot represents the cosine-modulated error distribution transformation, with dynamic skewness, scale, and tail heaviness adaptation over time. This modulation can handle cyclical or season structures of financial risk, such as those generated by day-of-week or month-of-year effects. The unified framework makes it possible to achieve more realism in clustering volatility, irregular outliers, and time-varying risk, which is desirable for risk forecasting, VaR and ES estimation, and cycle-sensitive financial choices. Its flexibility, however, calls for careful calibration to avoid overfitting in storage when there islimited strong empirical cyclicality.

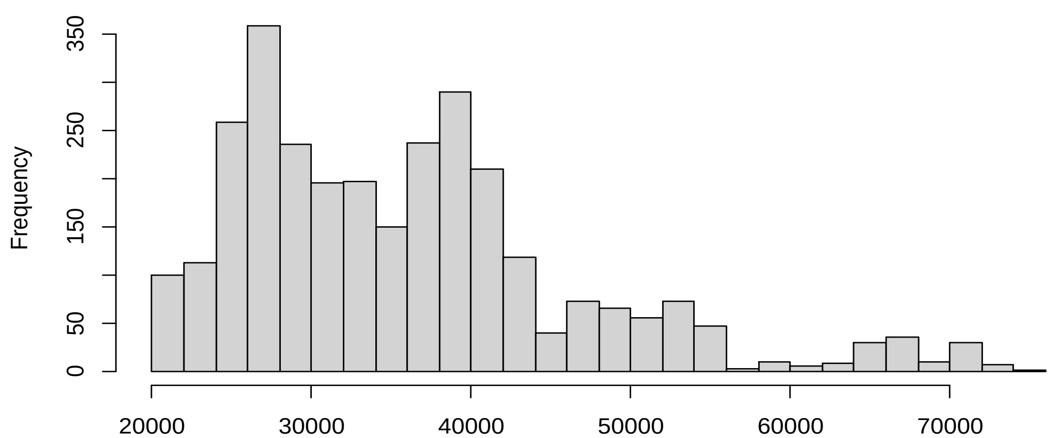

Figure 5: Plot of Nigeria Stock Exchange

Figure 5 reveals that NSE index values are mostly clustered between 30,000 and 40,000, with higher values having less frequencies. The histogram shows a right-skewed distribution,

indicating a wide spread and significant volatility in stock prices over time, ranging from approximately 20,000 to 70,000. The presence of multiple peaks in the plot suggests distinct market regimes and alternating periods of high and low volatility. This observed volatility pattern points to potential structural disruptions or shifts in market dynamics
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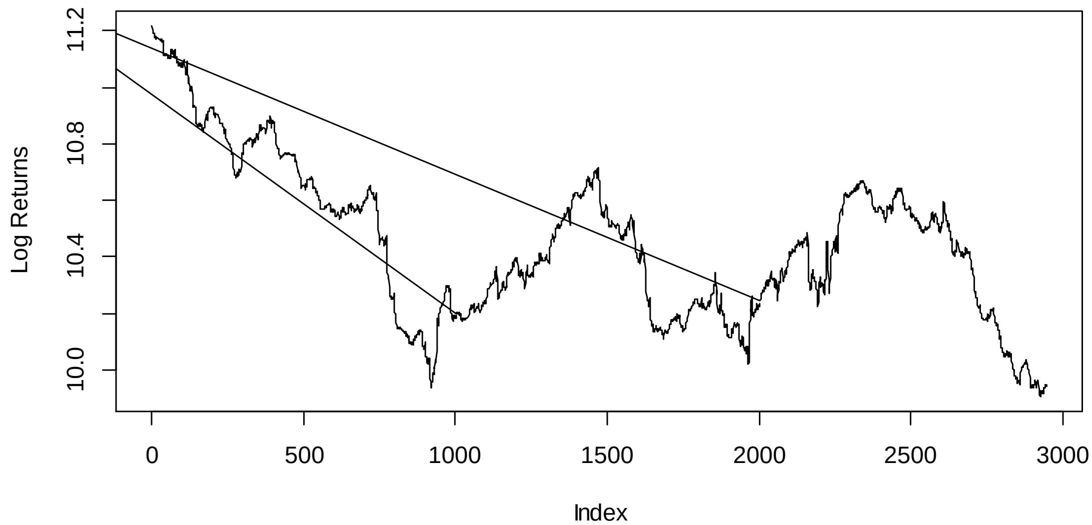

Figure 6: Plot of Nigeria Stock Exchange Price Log return

Figure6 illustrates the volatility dynamics of the Nigerian Stock Exchange (NSE), revealing distinct phases of upward and downward trends in log returns. A general pattern of declining returns is observed over the study period. The presence of volatility clustering periods of calm followed by periods of heightened fluctuations confirms the heteroskedastic nature of the series and supports the use of GARCH models. Sharp declines and subsequent recoveries may reflect shifts in investor sentiment, policy changes, or external economic shocks. The absence of a long-term trend suggests return stationarity, consistent with financial theory. These findings underscore the importance of employing advanced econometric models, such as the GARCH family with non-Gaussian innovations, to capture the persistence and structure of volatility in the NSE.

Table 2: Descriptive statistics for the Nigeria Stock Exchange

Min.	Max	1st Qu	Median	Mean	3rd Qu	skewness	kurtosis
20084	74289	27539	34239	35906	40870	1.14617	1.32576

Table2 reveals that, the Nigeria Stock Exchange data are right-skewed, ranging from a minimum of 20,084 to a maximum of 74,289, indicating a moderate degree of dispersion. The mean (35,906) is greater than the median (34,239), which indicates the positive skew, and one can observe that some extreme values are pushing the average up. The first quartile (27,539) and third quartile (40,870) indicate that the majority of the data points fall between these. A skewness of 1.14617 verifies the right-skewed nature, and a kurtosis of 1.32576 shows that it is platykurtic, i.e., the data are lighter-tailed and flatter compared to a normal distribution.

Table 3: Stationary test (ADF)

Test Statistics = -12.809	0.01*
Critical value = -3.43	0.05**

Table 2 reveals that the Augmented Dickey-Fuller test provided a test statistic of –12.809 and a p-value of 0.01*. Since the test statistic (–12.809) is less than the critical values (-3.45) with p-value less than 0.05, we reject the null hypothesis of a unit root. This means that the time series does not have a unit root and is therefore stationary

Applications of Real Data

This section contains the results of empirical data set of Nigeria Stock Exchange of three GARCH models GARCH (1,1), GARCH (1,2), and GARCH (2,1) with Cosine Exponentiated Generalized t (CEGST) error innovations Student's error innovations distribution. The models are compared in terms of significant statistical tests, volatility parameters, and predictive accuracy .The models are evaluated based on AIC, BIC, MAE, MSE, and RMSE.to choice the best fit model.

Table 4: GARCH model with Cosine Exponentiated Generalized Students t Error Innovation

PARAMTER	GARCH (1,1)	GARCH (1,2)	GARCH (2,1)
ARCH LM-test	Test Statistics=128.5 Pvalue==0.000	Test Statistics=331.42 Pvalue=1.18 X10^{-11}	Test Statistics=160.89 Pvalue==0.000
Box-Ljung tes	Test Statistics=162.2 Pvalue==0.000	Test Statistics=267.13 Pvalue=0.000	Test Statistics=91.059 Pvalue=3.33 X 10^{-15}
μ	0.00043	0.00014	0.00016
ω	0.00910	0.00058	0;0008
α 1	0.03851	0.27422	0.18044
α 2	-	-	0.01843
β1	0.65634	0.08295	0.08816
β2	-	0.08619	-
γ	0.00869	0.00002	0.00072
δ	0.30786	0.01788	0.01756
λ	0.12415	0.03293	0.49345
AIC	-31193.32	-46660.65	-46717.62
BIC	-31145.42	-46606.75	-46663.72
MAE	0.00637	0.00636	0.00636
MSE	0.0000093	0.000093	0.0000931
RMSE	0.00965	0.00965	0.00965

Table 4 presents the estimation results of the GARCH(1,1), GARCH(1,2), and GARCH(2,1) with cosine-exponentiated generalized Student's t (CEGST) error innovations (innovations). All the models were accepted by the Augmented Dickey-Fuller (ADF) test (test statistic = 0.01), lending support to the series of returns' stationarity. The ARCH LM test statistic was significant for all the models, and it was the largest for the GARCH(1,2) model at 331.42, indicating a better modeling of volatility using more lags. Similarly, the Box-Ljung test verified that all model specifications were appropriate, with the highest structure of autocorrelation for GARCH(1,2) being 267.13.Parameter estimates revealed almost zero values of the mean equation coefficients, as in stylized facts for financial returns.The coefficient for ARCH(1) (α_1) was most prominent in GARCH(1,2) (0.27422), signifying a higher response to past shocks, whereas GARCH(2,1) employed two ARCH terms for a more complex response of volatility. GARCH(1) coefficients (β_1) indicated high volatility persistence in GARCH(1,1) (0.65634), while GARCH(1,2) and

GARCH(2,1) indicated more distributed persistence due to additional lags.

Model selection using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) indicated GARCH(2,1) as the best-fitting model (AIC = -46,717.62; BIC = -46,663.72), followed by GARCH(1,2). GARCH(1,1) was the most poorly fitting model (AIC = -31,193.32). The fit of predictive performance, as reflected in MAE, MSE, and RMSE, was similar among models and, by extension, reflective of equal predictive accuracy given structural differences. In most cases, the best-fitting model was the GARCH(2,1) model with CEGST innovations, reflecting the effect of superior error innovation structure in capturing the non-Gaussian characteristics of financial returns. Future research can use other forms of distribution shapes, multivariate models, or regime-switching models for further robustness and predictive accuracy.

DISCUSSION

Both simulation outcomes and empirical findings sufficiently validate the efficiency of the proposed Cosine Exponentiated Generalized Student's t (CEGST) distribution as a sound innovation form for GARCH-type volatility models. They corroborate and supplement existing literature in identifying the importance of error innovation specification refinement in financial time series modeling.

Some recent studies have discovered the limitation of the typical GARCH models with normal or standard Student's t innovations in terms of modeling real-world features such as fat tails, skewness, and volatility clustering (Alotaibi & Hammoudeh, 2022; Asai & McAleer, 2020). Our study contributes to this body of literature by showing that the addition of cosine periodic transformations on the basis of cosine to heavy-tailed distributions enhances the ability of GARCH models to accommodate cyclical and asymmetric patterns of volatility, especially in emerging markets data. This is significant in the accurate modeling of financial return series whose repetitive shocks and seasonality features are common but poorly captured in standard specifications (Zhang & Zhang, 2021).

Moreover, the better simulation performance of the GARCH(1,2) model and the better empirical performance of the GARCH(2,1) model are in line with more general results in the literature that extra lags yield benefits for volatility modeling (Chen et al., 2023; Obaid et al., 2021). These configurations better describe complicated patterns of volatility and offer a better fit as well as predictive ability, particularly in persistence as well as regime change susceptible settings.

Most importantly, our findings also resonate more broadly with calls for methodological innovation in financial econometrics. For example, Nguyen and Park (2023) decry the strictness of assumptions in classical GARCH models and espouse hybrid versions that unite functional transformation and non-standard distributions. Our proposed CEGST innovation speaks directly to this, a flexible yet analytically tractable framework that permits both periodicity and heavy tails. In doing so, it offers a more informative tool for modeling volatility in cases such as the Nigerian Stock Exchange, whose return distributions are likely to deviate significantly from Gaussian assumptions due to structural breaks and macroeconomic shocks (Chung, 2024).

CONCLUSION

This article suggests Cosine Exponentiated Generalized Student's t (CEGST) distribution as a new and efficient new form of GARCH-type volatility models. Through simulation and empirical validation, establishes GARCH(1,2) and GARCH(2,1) models under CEGST error innovations as efficient volatility modeling tools for the Nigerian Stock Exchange. The GARCH(1,2) model is the best in simulation result, and the GARCH(2,1) is the best in the empirically ifindings with standard statistical requirements. These specifications are to be preferred over standard GARCH models with general Student's t or normal error innovations because they have a better potential to replicate fat tails, skewness, and time-varying volatility dynamics.

The findings emphasize the significance of having the suitable error innovation distributions in facilitating improvement both in the in-sample fit as well as out-of-sample forecast accuracy, particularly in non-Gaussian, heteroscedastic financial environments. The study therefore advances the theoretical and empirical frontiers of volatility modeling, particularly for emerging markets subject to structural breaks, cycle behaviour and tail risk.

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