



Research Article

Formulation of Generalized Hillslope Leakage-Dependent Model for Groundwater Flow in N-Aquifer System within a Sedimentary Basin

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ABSTRACT

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In this study, the recently derived Hillslope Leakage-dependent (HL-D) models for groundwater flow in leaky three-aquifer system have been generalized to obtain Generalized Hillslope Leakage-dependent, GHL-D equation that model groundwater flow in leaky n-aquifer system. This generalization extends the capability of Hillslope Leakage-dependent (HL-D) equations to modeling groundwater flow in any number of aquifers (n-aquifer system) of all configurations within a sedimentary basin. The formulated GHL-D Model was tested on some aquifer types and the results have shown that it can be used to obtain groundwater flow models for both hillslope (sloping) and horizontal single aquifer (unconfined and confined) as well as for multi-aquifer system containing any number of aquifers within a sedimentary basin. This unique feature of GHL-D equation to model most aquifers types and configurations makes it a more elegant groundwater flow modeling tool with potentials for wider applications than most of the existing groundwater flow models. The GHL-D Model, when use for simulations of groundwater flow in single or multi-aquifer system, will have the capability of giving better understanding of the groundwater flow dynamics in such systems. This is important because most of the sedimentary basins of the world are multi-aquifer in nature and accurate knowledge of groundwater flow within them is important to geoscientists in determining long time yield and water sustainability in such geological structures

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INTRODUCTION

Groundwater worldwide contributes about 20% of the people's fresh water and groundwater basins are important long-term storage reservoirs which in semi-arid and arid countries often constitute the only perennial water resource (Wolfgang *et al.*, 2007). Aquifers are subsurface geological structures that hold groundwater and from which significant quantities of water can be extracted (Salako and Adepelumi, 2018). General phenomenal of falling water tables, drying wetlands and desert encroachment and general deterioration of water quality are some of the evidences of lack of groundwater sustainability being experienced all over the world (Bansal, 2012). To solve these problems and a variety of others is the main purpose for modeling flow and transport in groundwater systems. Geoscientists are using models as analytical tools for different purposes such as to predict the effects of hydrological changes, groundwater abstraction or irrigation developments and planning various water management programmes for urban areas (Suárez, 2003; Giere, 2015). General equation to study groundwater flow dynamics within uniformly sloping hillslopes was first derived by Boussinesq in 1877 (Boussinesq, 1877) and since that time till now, various forms of this equation for subsurface flow on a sloping base (or hillslope flow) have been studied extensively. Most of the researches in the existing literatures like those of Koussis and Akylas (2011), Troch *et al.* (2013), and Tetzlaff *et al.* (2014) have applied hillslope models to single aquifer system. In these applications, no-flow boundary condition has been imposed on the base of the aquifer, thereby, foreclosing the possibility of leakage especially in multi-aquifer system. Also, applications and analysis of the Boussinesq's equations were done by Abdellaoui *et al.*,

(2015) and Telyakovskiy *et al.* (2016) while Djida *et al.* (2016) and Atangana and Alkhatani (2016) demonstrated the application of fractional partial differential equations (fPDEs) to groundwater flow problems. Although the results of various applications of Boussinesq's equations have been remarkably accurate yet the assumption of zero flow across the confining layers limited the applications of these equations. A number of field studies, complemented by a handful of modeling studies, have examined the importance of leakage in a hillslope context (Montgomery *et al.*, 2002; Uchida *et al.*, 2003; Tromp-van *et al.*, 2007; Ebel *et al.*, 2008; Graham *et al.*, 2010; Troch *et al.*, 2013; Abdellaoui *et al.*, 2015). Baker and Hemker (2002) derived groundwater flow equation in layered, anisotropic aquifer but the result is not applicable to aquifer system consisting aquifers separated by aquitards. Also, in order to study leakages through aquitards separating aquifers in a two-aquifer system, Broda *et al.* (2008) added a leakage term to a version of Boussinesq equation and used it to explore the sensitivity of the resulted model to a range of constant and variable leakage rate. However, the underlying aquifer (aquifer 2) was considered horizontal which made the resulting model not wholly hillslope in nature. In an attempt to obtain a wholly hillslope model for groundwater flow in multi-aquifer system in Boussinesq context, Sonloye *et al.* (2022) recently derived Hillslope Leakage-dependent (HL-D) groundwater flow models for leaky three-aquifer system. However, these models are not applicable beyond the three-aquifer system. Since most of the sedimentary basins of the world are multi-aquifer in nature (Kruseman and DeRidder, 1990) and such geological structures are important long-term storage reservoirs especially in semi-arid and arid countries where it is usually the only

perennial water resource (Wolfgang *et al.*, 2007), accurate knowledge of leakages and groundwater flow within multi-aquifer systems are important in determining the water sustainability in such environments. This paper will, therefore, generalize the Hillslope Leakage-dependent (HL-D) models for groundwater flow in three-aquifer system to obtain a Generalized Hillslope Leakage-dependent (GHL-D) model for groundwater flow in any number of aquifers (n-aquifer system) of all configurations within a sedimentary basin. This is important because the introduction of leakages into Boussinesq equation and its extension to model

$$\frac{\partial h_1}{\partial t} = \frac{k_1 \cos \varphi}{f} \frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_1 \sin \varphi}{f} \frac{\partial h_1}{\partial x} + \frac{R}{f} + \frac{k_{v1}(h_2 - h_1)}{fd_1} \quad (1)$$

$$\frac{\partial h_2}{\partial t} = \frac{T_2 \cos \varphi}{S_{r2}} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v1}}{S_{r2}} \left(\frac{h_2 - h_1}{d_1} \right) + \frac{k_{v2}}{S_{r2}} \left(\frac{h_3 - h_2}{d_2} \right) \quad (2)$$

$$\frac{\partial h_3}{\partial t} = \frac{T_3 \cos \varphi}{S_{r3}} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v2}}{S_{r3}} \left(\frac{h_3 - h_2}{d_2} \right) \quad (3)$$

The upper unconfined aquifer, middle confined aquifer and lower confined aquifer will hereafter be referred to as aquifer 1, aquifer 2 and aquifer 3 respectively. Equations (1), (2) and (3) are not applicable beyond the three-aquifer system for which it was derived. Since it is possible that a sedimentary basin can contain higher or smaller number of aquifers than three aquifers, a more generalized model will be :

$$\frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) = \frac{1}{2} \frac{\partial^2 h_1^2}{\partial x^2} \quad (4)$$

Substituting (4) into (1) and rearranging, (1) becomes:

$$\frac{\partial h_1}{\partial t} = \frac{k_1 \cos \varphi}{2f} \frac{\partial^2 h_1^2}{\partial x^2} + \frac{k_1 \sin \varphi}{f} \frac{\partial h_1}{\partial x} + \frac{R}{f} + \frac{k_{v1}(h_2 - h_1)}{fd_1}$$

groundwater flow in multi-aquifer system will lead to better understanding of the groundwater flow dynamics in such geological structures and it is crucial for estimating long time yields of aquifers.

Leakage-dependent Groundwater Flow Equations in Terms of Aquifer fluxability

Sonloye *et al.* (2022) derived the Hillslope Leakage-dependent (HL-D) groundwater flow in a uniform width homogeneous and isotropic upper unconfined aquifer, middle semi-confined aquifer and lower semi-confined aquifer in a three-aquifer system in term of groundwater head, *h* as:

needed to handle such complexity. To do this, the three equations modeling groundwater flow within the three-aquifer system will have to be generalized for n number of aquifers within a sedimentary basin. The first step in this direction is to transform and write each of the three groundwater flow equations above in term of a parameter called aquifer fluxability, ϕ . Recalling the differentiation identity

$$\frac{\partial h_1}{\partial t} = \frac{\cos \varphi}{f} \frac{\partial^2 \left(\frac{1}{2} k_1 h_1^2 \right)}{\partial x} + \frac{\sin \varphi}{f} \frac{\partial (k_1 h_1)}{\partial x} + \frac{R}{f} - \frac{k_{v1}}{fd_1} h_1 + \frac{k_{v1}}{fd_1} h_2$$

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} + B_1 \frac{\partial \phi_1}{\partial x} - C_1 \phi_1 + E_1 \phi_2 + G_1 \tag{5}$$

where $\phi_1 = \frac{1}{2} k h_1^2$, $\phi_2 = k_2 D_2 h_2$, $h_1 = \frac{2\phi_1}{k_1 h_1}$, $h_2 = \frac{\phi_2}{k_2 D_2}$, \bar{h}_1 is the average

hydraulic head in aquifer 1 and it is given by Werner as cited by Teloglou and Bansal (2012) as $\bar{h}_1 = \frac{h_0 - h_t}{2}$, h_0 is the initial hydraulic head while h_t is the head at the present time,

$A_1 = \frac{T_1 \cos \varphi}{2f}$, $B_1 = \frac{k_1 \sin \varphi}{f}$, $C_1 = \frac{k_{v1}}{fd_1}$, $E_1 = \frac{k_{v1} T_1}{2fd_1 T_2}$, $G_1 = \frac{RT_1}{2f}$; $T_1 = k_1 \bar{h}_1$ and $T_2 = k_2 \bar{D}_2$ are the transmissivity in aquifer 1 and 2 respectively.

Also, the hillslope leakage-dependent groundwater flow equation in aquifer 2 can be transformed by rearranging (2) as:

$$\frac{\partial h_2}{\partial t} = \frac{\cos \varphi}{S_{t2}} \frac{\partial^2 (k_2 D_2 h_2)}{\partial x^2} + \frac{k_{v1}}{S_{t2} d_1} h_1 - \frac{1}{S_{t2}} \left(\frac{k_{v1}}{d_1} + \frac{k_{v2}}{d_2} \right) h_2 + \frac{k_{v2}}{S_{t2} d_2} h_3 \tag{6}$$

If we substitute $\phi_2 = k_2 D_2 h_2$, $\phi_3 = k_3 D_3 h_3$, $h_3 = \frac{\phi_3}{k_3 D_3}$ as well as the values of h_1 and h_2

as defined in (5) into (6) and multiplying the result through by $k_2 \bar{D}_2$, we will obtain:

$$\frac{\partial \phi_2}{\partial t} = A_2 \frac{\partial^2 \phi_2}{\partial x^2} + B_2 \phi_1 - C_2 \phi_2 + E_2 \phi_3 \tag{7}$$

where $A_2 = \frac{T_2 \cos \varphi}{S_{t2}}$, $B_2 = \frac{2T_2 k_{v1}}{T_1 d_1 S_{t2}}$, $C_2 = \frac{1}{S_{t2}} \left(\frac{k_{v1}}{d_1} + \frac{k_{v2}}{d_2} \right)$, $E_2 = \frac{T_2 k_{v2}}{T_3 d_2 S_{t2}}$, $T_3 = k_3 \bar{D}_3$ is the

transmissivity in aquifer 3. The equation for groundwater flow in aquifer 3 can be obtained by transforming (3) following the same procedures used in obtaining (7) as follows:

$$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3 \cos \varphi}{S_{t3}} \frac{\partial^2 h_3}{\partial x^2} + \frac{k_3 \bar{D}_3 k_{v2}}{k_2 \bar{D}_2 d_2 S_{t3}} \phi_2 - \frac{k_{v2}}{d_2 S_{t3}} \phi_3$$

$$\frac{\partial \phi_3}{\partial t} = A_3 \frac{\partial^2 \phi_3}{\partial x^2} + B_3 \phi_2 - C_3 \phi_3 \tag{8}$$

where $A_3 = \frac{T_3 \cos \varphi}{S_{t3}}$; $B_3 = \frac{T_3 k_{v2}}{T_2 d_2 S_{t3}}$; $C_3 = \frac{k_{v2}}{d_2 S_{t3}}$

Note that the aquifer fluxability, ϕ_1 , ϕ_2 and ϕ_3 in each of the equations (5), (7) and (8) fluxability, ϕ can be defined as the volume of water that can be transmitted through a

respectively has the same unit as groundwater volumetric flux (m^3/day). Therefore, aquifer given point within an aquifer per unit time. Equations (5), (7) and (8) are the derived

hillslope leakage-dependent (HL-D) groundwater flow equations, in term of aquifer fluxability, ϕ within a uniform width homogeneous and isotropic upper unconfined aquifer (aquifer 1), middle semi-confined aquifer (aquifer 2) and lower semi-confined aquifer (aquifer 3) respectively within a three-aquifer system.

Generalization of Hillslope Leakage-dependent (HL-D) models for three-aquifer system into n-aquifer System

$$\frac{\partial \phi_n}{\partial t} = A_n \frac{\partial^2 \phi_n}{\partial x^2} + B_n \phi_{n-1} - C_n \phi_n + E_n \phi_{n+1} \tag{9}$$

Since (9) is obtained from groundwater flow equation in aquifer 2, which is a confined aquifer, then all the parameters in it are mainly for confined aquifer. Hence, it cannot adequately represent groundwater flow in unconfined aquifer. For (9) to be able to represent groundwater flow in all aquifer

$$\frac{\partial \phi_n}{\partial t} = A_n \frac{\partial^2 \phi_n}{\partial x^2} + B_n \phi_{n-1} - C_n \phi_n + E_n \phi_{n+1} + P \left(B_n \frac{\partial \phi_n}{\partial x} + G_n \right) \tag{10}$$

where ϕ_n is the aquifer fluxability in nth aquifer within an n-aquifer system and $n=1,2,3,\dots$. For an unconfined aquifer, $\phi_n = \frac{1}{2} k_n h_n^2$ but in a confined aquifer, $\phi_n = k_n D_n h_n$. Also, at $n=1$, ϕ_{n-1} becomes $\phi_0 = 0$ in (10). This because there is no aquifer before the first aquifer. For the

$$0 \leq P \leq 1, \quad P = \begin{cases} 1 & \text{unconfined aquifer} \\ 0 & \text{confined aquifer} \end{cases}$$

Equation (10) is the generalized hillslope leakage-dependent groundwater flow model in homogeneous and isotropic n-aquifer system. It can be shortened as GHL-D Model. By comparing coefficients in (5), (7) and (8), A_n, B_n, C_n and E_n

can be obtained as: $A_n = \frac{J_n T_n \cos \varphi}{g_n}$, where $J_n = \begin{cases} 0.5 & \text{unconfined aquifer} \\ 1 & \text{confined aquifer} \end{cases}$,

$$T_n = \begin{cases} k \bar{h}_n & \text{unconfined aquifer} \\ k D_n & \text{confined aquifer} \end{cases} \text{ and } g_n = \begin{cases} f & \text{unconfined aquifer} \\ S_{t(n)} & \text{confined aquifer} \end{cases}; \quad B_n = \frac{k_n \sin \varphi}{f} \text{ for}$$

unconfined aquifer while $B_n = \frac{T_n k_{v(n-1)}}{\beta_n T_{(n-1)} d_{(n-1)} S t_n}$ for confined aquifer;

The Hillslope Leakage-dependent (HL-D) models for groundwater flow in aquifers within a three-aquifer system can now be generalized for n-aquifer system by using equation (7). This is because equation (7) consists of ϕ_1, ϕ_2 and ϕ_3 . It can, therefore, be used as the basis for the model generalization. Equation (7) can be generalized for nth aquifer within a multi-aquifer system as:

types, including unconfined aquifer, parameters from unconfined aquifer must be present in it. The second term and the last term on the right side of equation (5) only exist in an unconfined aquifer. Therefore, these two terms are added to (9) with a constant, P and it becomes:

last aquifer, $n=l$, then, ϕ_{l+1} in (10) becomes $\phi_{l+1} = 0$. Where l is the last aquifer at the bottom of the multi-aquifer system. This is because there is no other aquifer after the last one at the bottom of the multi-aquifer system. P is a constant called the ‘‘unsaturated zone diffusion coefficient’’ and its range is

$$C_n = \frac{1}{g_n} \left[\frac{k_{v(n-1)}}{d_{(n-1)}} + \frac{k_{vn}}{d_n} \right] \text{ and } E_n = \frac{T_n k_{v(n)}}{g_n T_{(n+1)} d_n} \text{ where } T_{(n+1)} \text{ is the transmissivity of the aquifer}$$

after (below) the nth aquifer and $G_1 = \frac{Rk_1 h_1}{2f}$. If the first aquifer, n=1, is an unconfined

aquifer, $\beta_2=0.5$, otherwise $\beta_n=1$, $T_{(n-1)}$ is the transmissivity of the aquifer before (on top) of the nth aquifer. If n=1, then $k_{v(n-1)}$ becomes: $k_{v0} = 0$ because the vertical hydraulic conductivity of the confining layer on top of a completely confined aquifer is approximately zero. Also, $D_0 = k_0 = d_0 \approx 1$. The choice of unity for the value of the upper confining layer thickness, d_0 , the hydraulic conductivity, k_0 and thickness of the formation above the first aquifer, D_0 is a matter of convenience as this value neither increased nor decreased the value of B_1 . Consequently, the general groundwater flow within the first confined aquifer is not affected.

Derivation of Groundwater Flow Model in some Aquifer Types and Configurations from GHL-D Model

The possibility of using GHL-D Model to derive groundwater flow models of other aquifer types and configurations will be a good proof that it is indeed a generalized model. Therefore, GHL-D Model will be used to obtain groundwater flow equations in single unconfined and confined aquifers as well as in multi-aquifer systems containing higher number of aquifers than three and with configuration different from that of the three-aquifer system from which it was derived.

Derivation of groundwater flow equation in an unconfined aquifer from GHL-D groundwater flow model

Generalized hillslope leakage-dependent (GHL-D) groundwater flow in n-aquifer system has been obtained in this paper as (10):

$$\frac{\partial \phi_n}{\partial t} = A_n \frac{\partial^2 \phi_n}{\partial x^2} + B_n \phi_{n-1} - C_n \phi_n + E_n \phi_{n+1} + P \left(B_n \frac{\partial \phi_n}{\partial x} + G_n \right)$$

For a sloping single unconfined aquifer, where n=1 and P=1, GHL-D Model above reduces to (5). Since no-flow boundary will

have to be imposed on the aquifer base for a single unconfined aquifer, the leakage terms will disappear, and (5) will reduce to

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} + B_1 \frac{\partial \phi_1}{\partial x} + G_1 \tag{11}$$

where $\phi_1 = \frac{1}{2} k h_1^2$. A_1, B_1, G_1 can be obtained from A_n, B_n, G_n , defined for (10), as

$$A_1 = \frac{T_1 \cos \varphi}{2f}, T_1 = k_1 \overline{h_1}, B_1 = \frac{k_1 \sin \varphi}{f} \text{ and } G_1 = \frac{Rk_1 h_1}{2f}$$

left-hand side of (4) are substituted into (11) and multiply the result through by $\frac{2}{k_1 h_1}$, the

groundwater flow equation in a single hillslope (sloping) unconfined aquifer in term of h will be obtained as:

$$\frac{\partial h}{\partial t} = \frac{k \cos \varphi}{f} \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{k \sin \varphi}{f} \frac{\partial h}{\partial x} + \frac{R}{f} \tag{12}$$

For a horizontal unconfined aquifer, where $\varphi = 0$, equation (12) reduces to the conventional groundwater flow equation in an unconfined single aquifer, which is:

$$\frac{\partial h}{\partial t} = \frac{k}{f} \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{R}{f} \tag{13}$$

Note that n=1 has been left out of (12) because the aquifer is only one and there is, therefore, no need for numbering its head.

Derivation of groundwater flow equation in a confined aquifer from GHL-D groundwater flow model

For a single confined aquifer, where n=1, P=0 and $k_{v0}=0$, (10) reduces to:

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} - C_1 \phi_1 + E_1 \phi_2 \tag{14}$$

Also, in a single confined aquifer, where no-flow boundary condition has to be imposed on the aquifer base, the leakage terms will disappear and (14) will reduce to:

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} \tag{15}$$

where $\phi_1 = k_1 \overline{D}_1 h_1$. A_1 can be derived from $A_n = \frac{J_n T_n \cos \varphi}{g_n}$ in (10) as $A_1 = \frac{T_1 \cos \varphi}{St_1}$ and

$T_1 = k_1 \overline{D}_1$ is the transmissivity of the confined aquifer. If the values of ϕ_1 and A_1 are substituted into (15) and the result is

$$\frac{\partial h}{\partial t} = \frac{k \cos \varphi}{St} \frac{\partial^2 h}{\partial x^2} \tag{16}$$

In a horizontal confined aquifer, where $\varphi = 0$, (16) reduces to the conventional

$$\frac{\partial h}{\partial t} = \frac{k}{St} \frac{\partial^2 h}{\partial x^2} \tag{17}$$

Hence, it has been proven that groundwater flow equations for both hillslope and horizontal single confined and unconfined aquifers can be derived from GHL-D Model.

Derivation of Groundwater Flow Models in Multi-Aquifer System Containing More Than Three Aquifers from GHL-D Groundwater Flow Model

The fact that GHL-D Model was derived from a three-aquifer system containing an unconfined aquifer on top of two confined aquifers is an indication that it can be used to model groundwater flow in three aquifer system. Since GHL-D model is a generalized

rearranged, the groundwater flow equation in a single hillslope confined aquifer will be obtained in term of h as:

groundwater flow equation in a single confined aquifer, which is:

model for all hillslope aquifers, it is expected that it should be able to model groundwater flow in multi-aquifer systems containing higher number of aquifers than three and of configuration different from that of the three-aquifer system from which it was derived. To prove this, equations of groundwater flow in a five-aquifer system will be derived from GHL-D model for two different configurations of five-aquifer system.

Case 1: One (1) unconfined aquifer and four (4) confined aquifers

For the first leaky unconfined aquifer, where n=1 and P=1, the groundwater flow equation

for aquifer 1 can be obtained from (10) as (5)

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} + B_1 \frac{\partial \phi_1}{\partial x} - C_1 \phi_1 + E_1 \phi_2 + G_1$$

If the values of ϕ_1 , A_1 , B_1 , G_1 , T_1 defined in (5) and the left-hand side of (4) are substituted into (5) and multiply the result

which is restated below as:

through by $\frac{2}{k_1 h_1}$, the groundwater flow equation in the upper leaky unconfined hillslope (sloping) aquifer in term of h will reduce to (1) which is restated here as:

$$\frac{\partial h_1}{\partial t} = \frac{k_1 \cos \varphi}{f} \frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_1 \sin \varphi}{f} \frac{\partial h_1}{\partial x} + \frac{k_{v1}(h_2 - h_1)}{f d_1} + \frac{R}{f}$$

For a horizontal aquifer, where $\varphi = 0$, equation (1) becomes:

$$\frac{\partial h_1}{\partial t} = \frac{k_1}{f} \frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_{v1}(h_2 - h_1)}{f d_1} + \frac{R}{f} \quad (18)$$

The groundwater flow in the second confined aquifer, where $n=2$ and $p=0$, is obtained from (10) as (7):

$$\frac{\partial \phi_2}{\partial t} = A_2 \frac{\partial^2 \phi_2}{\partial x^2} + B_2 \phi_1 - C_2 \phi_2 + E_2 \phi_3$$

where $\phi_2 = k_2 D_2 h_2$. A_2 , B_2 , C_2 , and E_2 are obtained, from the definitions of A_n , B_n , C_n and E_n

in (10), as $A_2 = \frac{T_2 \cos \varphi}{S t_2}$, $B_2 = \frac{2 T_2 k_{v(1)}}{T_1 d_1 S t_2}$, $C_2 = \frac{1}{S_{r2}} \left(\frac{k_{v1}}{d_1} + \frac{k_{v2}}{d_2} \right)$, $E_2 = \frac{T_2 k_{v2}}{T_3 d_2 S_{r2}}$

respectively and $T_3 = k_3 \overline{D_3}$. If the values of ϕ_2 , A_2 , B_2 , C_2 , E_2 are substituted into (7) and dividing it through by $K_2 D_2$, (7) reduces to (2) which is:

$$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2 \cos \varphi}{S t_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 S t_2} + \frac{k_{v(2)}(h_3 - h_2)}{S t_2 d_2}$$

In a horizontal leaky confined aquifer, equation (2) will reduce to:

$$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2}{S t_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 S t_2} + \frac{k_{v(2)}(h_3 - h_2)}{S t_2 d_2} \quad (19)$$

The groundwater flow equation in the third leaky confined aquifer, where $n=3$ and $P=0$, is obtained from (10) as:

$$\frac{\partial \phi_3}{\partial t} = A_3 \frac{\partial^2 \phi_3}{\partial x^2} + B_3 \phi_2 - C_3 \phi_3 + E_3 \phi_4 \quad (20)$$

where $\phi_3 = k_3 D_3 h_3$ and $\phi_4 = k_4 D_4 h_4$. A_3, B_3, C_3 and E_3 are obtained, from the definitions of $A_n,$

$$B_n, C_n \text{ and } E_n \text{ in (10), as } A_3 = \frac{T_3 \cos \varphi}{St_3}, B_3 = \frac{T_3 k_{v(2)}}{T_2 d_2 St_3}, C_3 = \frac{1}{St_3} \left[\frac{k_{v(2)}}{d_2} + \frac{kv_3}{d_3} \right], E_3 = \frac{T_3 k_{v(3)}}{St_3 T_4 d_3}$$

respectively. If the values of $\phi_3, A_3, B_3, C_3, E_3$ and ϕ_4 are substituted into (20) and dividing through by $K_3 D_3$, it reduces to:

$$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3 \cos \varphi}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3} \quad (21)$$

In a horizontal leaky confined aquifer, (21) becomes:

$$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3} \quad (22)$$

The groundwater flow equation in the fourth leaky confined aquifer within the 5-aquifer system, where $n=4$ and $P=0$, is obtained from the GHLD model as:

$$\frac{\partial \phi_4}{\partial t} = A_4 \frac{\partial^2 \phi_4}{\partial x^2} + B_4 \phi_3 - C_4 \phi_4 + E_4 \phi_5 \quad (23)$$

The coefficients A_4, B_4, C_4 and E_4 are obtained, from the definitions of A_n, B_n, C_n and E_n in (10), as

$$A_4 = \frac{T_4 \cos \varphi}{St_4}, B_4 = \frac{T_4 k_{v(3)}}{T_3 d_3 St_4}, C_4 = \frac{1}{St_4} \left[\frac{k_{v(3)}}{d_3} + \frac{kv_4}{d_4} \right], E_4 = \frac{T_4 k_{v(4)}}{St_4 T_5 d_4} \text{ respectively and}$$

$\phi_5 = k_5 D_5 h_5$. If the values of $\phi_4, A_4, B_4, C_4, E_4$ and ϕ_5 are substituted into (23) and the result is divided through by $k_4 D_4$, (23) becomes:

$$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4 \cos \varphi}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4} \quad (24)$$

For a horizontal leaky confined aquifer, (24) becomes:

$$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4} \quad (25)$$

The groundwater flow model for the fifth leaky confined aquifer within the 5-aquifer system, where $n=5, P=0$ and $k_{v(5)}=0$, can be obtained from (10) as:

$$\frac{\partial \phi_5}{\partial t} = A_5 \frac{\partial^2 \phi_5}{\partial x^2} + B_5 \phi_4 - C_5 \phi_5 \quad (26)$$

The coefficients A_5, B_5 and C_5 are obtained from A_n, B_n, C_n , defined for (10), as $A_5 = \frac{T_5 \cos \varphi}{St_5}$,

$$B_5 = \frac{T_5 k_{v(4)}}{T_4 d_4 St_5}, C_5 = \frac{k_{v(4)}}{St_5 d_4}. \text{ It is worthy of note that } \phi_6 = 0 \text{ because there}$$

is no sixth aquifer in a 5-aquifer system. Also, since a 5-aquifer system only has four (4) aquitards separating its 5 aquifers and the base of the fifth aquifer is an aquiclude with

$k_v=0$, therefore, $k_{v(5)}=0$. If the values of ϕ_4, A_5, B_5, C_5 and ϕ_5 are substituted into (26) and dividing it through by $k_5 D_5$, (26) becomes:

$$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5 \cos \varphi}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5} \quad (27)$$

In a horizontal leaky confined aquifer, (27) becomes:

$$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5} \quad (28)$$

Case 2: 5 confined aquifers

The groundwater flow equation in upper leaky confined aquifer, where n=1 and P=0, is obtained from (10) as (14) which is restated here as:

$$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} - C_1 \phi_1 + E_1 \phi_2$$

where $\phi_1 = k_1 \overline{D_1} h_1$. For a confining top layer that is completely impermeable, $k_{v(0)}=0$ and $d_0=1$.

The coefficients A_1 , C_1 and E_1 are obtained, from the definitions of A_n , C_n and E_n for (10), as

$$A_1 = \frac{T_1 \cos \varphi}{St_1}, \quad C_1 = \frac{k_{v1}}{St_1 d_1} \text{ and } E_1 = \frac{T_1 k_{v(1)}}{St_1 T_2 d_1}. \text{ If } \phi_1 = k_1 \overline{D_1} h_1 \text{ and the values } A_1, C_1 \text{ and } E_1 \text{ are}$$

substituted into (14) and the result is divided through by $k_1 \overline{D_1}$, (14) becomes:

$$\frac{\partial h_1}{\partial t} = \frac{k_1 D_1 \cos \varphi}{St_1} \frac{\partial^2 h_1}{\partial x^2} + \frac{k_{v1}(h_2 - h_1)}{St_1 d_1} \quad (29)$$

Also, in a horizontal upper leaky confined aquifer, (29) reduces to:

$$\frac{\partial h_1}{\partial t} = \frac{k_1 D_1}{St_1} \frac{\partial^2 h_1}{\partial x^2} + \frac{k_{v1}(h_2 - h_1)}{St_1 d_1} \quad (30)$$

The groundwater flow in the second leaky confined aquifer, where n=2 and P=0, is obtained from (10) as:

$$\frac{\partial \phi_2}{\partial t} = A_2 \frac{\partial^2 \phi_2}{\partial x^2} + B_2 \phi_1 - C_2 \phi_2 + E_2 \phi_3 \quad (31)$$

where $\phi_2 = k_2 \overline{D_2} h_2$. A_2 , B_2 , C_2 , and E_2 are obtained as $A_2 = \frac{T_2 \cos \varphi}{St_2}$, $B_2 = \frac{2T_2 k_{v(1)}}{T_1 d_1 St_2}$,

$C_2 = \frac{1}{S_{r2}} \left(\frac{k_{v1}}{d_1} + \frac{k_{v2}}{d_2} \right)$ and $E_2 = \frac{T_2 k_{v2}}{T_3 d_2 S_{r2}}$ respectively while $T_3 = k_3 \overline{D_3}$. If the values of ϕ_2 ,

A_2 , B_2 , C_2 , E_2 are substituted into (31), dividing it through by $k_2 \overline{D_2}$ and rearranging, (31) reduces to:

$$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2 \cos \varphi}{St_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2} \quad (32)$$

For a horizontal leaky confined aquifer, (32) reduces to:

$$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2}{St_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2} \quad (33)$$

Following the same procedures used in obtaining (20), (21) and (22), the groundwater flow equation in the third hillslope leaky confined aquifer in term of ϕ , hillslope leaky confined aquifer in term of h and horizontal leaky confined aquifer in term h respectively, the GHL-D model also yields (20), (21) and (22) and are restated here as:

$$\frac{\partial \phi_3}{\partial t} = A_3 \frac{\partial^2 \phi_3}{\partial x^2} + B_3 \phi_2 - C_3 \phi_3 + E_3 \phi_4,$$

$$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3}{St_3 \cos^2 \varphi} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3},$$

$$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3}$$

Also, if the same procedures used in obtaining (23), (24) and (25) are followed, the groundwater flow equation in the fourth hillslope leaky confined aquifer in term of ϕ , hillslope leaky confined aquifer in term

of h and horizontal leaky confined aquifer in term h respectively, can be obtained from the GHL-D model which also yields (23), (24) and (25) respective and are restated here as:

$$\frac{\partial \phi_4}{\partial t} = A_4 \frac{\partial^2 \phi_4}{\partial x^2} + B_4 \phi_3 - C_4 \phi_4 + E_4 \phi_5,$$

$$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4}{St_4 \cos^2 \varphi} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$$

$$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$$

Similarly, following the same procedures used in obtaining (26), (27) and (28), the groundwater flow equation in the fifth hillslope leaky confined aquifer in term of ϕ , hillslope leaky confined aquifer in term

of h and horizontal leaky confined aquifer in term h respectively, can be obtained from the GHL-D model which still yields (26), (27) and (28) respective and are restated here as:

$$\frac{\partial \phi_5}{\partial t} = A_5 \frac{\partial^2 \phi_5}{\partial x^2} + B_5 \phi_4 - C_5 \phi_5,$$

$$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5}{St_5 \cos^2 \varphi} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$$

$$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$$

Hence, the derivation of HL-D models for 5-aquifer system and horizontal leakage-dependent 5-aquifer system of two different configurations from GHL-D model as well groundwater flow equations in hillslope and horizontal single confined and unconfined aquifers, is a proof that GHL-D model is indeed a generalized model that is leakage-dependent and, therefore, has the capacity to model groundwater flow in most aquifer types and configurations. The groundwater flow equations single aquifers and multi-aquifer systems derived from GHL-D model are summarized in table 1

Table 1: Summary of Groundwater flow equations for single aquifers and 5-aquifer system derived in terms of Aquifer fluxability, ϕ and head, h from GHL-D Model

AQUIFER TYPES	AQUIFER CONFIGURATIONS	GROUNDWATER FLOW EQUATIONS		
		(IN TERMS OF ϕ)	(IN TERMS OF h)	
			HILLSLOPE (SLOPING) AQUIFERS	HORIZONTAL AQUIFERS
SINGLE AQUIFER	SINGLE UNCONFINED AQUIFER	$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} + B_1 \frac{\partial \phi_1}{\partial x} + G_1$	$\frac{\partial h}{\partial t} = \frac{k \cos \varphi}{f} \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{k \sin \varphi}{f} \frac{\partial h}{\partial x} + \frac{R}{f}$	$\frac{\partial h}{\partial t} = \frac{k}{f} \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{R}{f}$
	SINGLE CONFINED AQUIFER	$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2}$	$\frac{\partial h}{\partial t} = \frac{k \cos \varphi}{St} \frac{\partial^2 h}{\partial x^2}$	$\frac{\partial h}{\partial t} = \frac{k}{St} \frac{\partial^2 h}{\partial x^2}$
MULTI-AQUIFER SYSTEMS	5-AQUIFER SYSTEM	$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} + B_1 \frac{\partial \phi_1}{\partial x} - C_1 \phi_1 + E_1 \phi_2 + G_1$	$\frac{\partial h_1}{\partial t} = \frac{k_1 \cos \varphi}{f} \frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_1 \sin \varphi}{f} \frac{\partial h_1}{\partial x} + \frac{k_{v1}(h_2 - h_1)}{fd_1} + \frac{R}{f}$	$\frac{\partial h_1}{\partial t} = \frac{k_1}{f} \frac{\partial}{\partial x} \left(h_1 \frac{\partial h_1}{\partial x} \right) + \frac{k_{v1}(h_2 - h_1)}{fd_1} + \frac{R}{f}$
	•1 Unconfined aquifer	$\frac{\partial \phi_2}{\partial t} = A_2 \frac{\partial^2 \phi_2}{\partial x^2} + B_2 \phi_1 - C_2 \phi_2 + E_2 \phi_3$	$\frac{\partial h_2}{\partial t} = \frac{T_2 \cos \varphi}{St_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2}$	$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2}{St_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2}$
	•4 confined aquifers	$\frac{\partial \phi_3}{\partial t} = A_3 \frac{\partial^2 \phi_3}{\partial x^2} + B_3 \phi_2 - C_3 \phi_3 + E_3 \phi_4$	$\frac{\partial h_3}{\partial t} = \frac{T_3 \cos \varphi}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3}$	$\frac{\partial h_3}{\partial t} = \frac{T_3}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3}$
		$\frac{\partial \phi_4}{\partial t} = A_4 \frac{\partial^2 \phi_4}{\partial x^2} + B_4 \phi_3 - C_4 \phi_4 + E_4 \phi_5$	$\frac{\partial h_4}{\partial t} = \frac{T_4 \cos \varphi}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$	$\frac{\partial h_4}{\partial t} = \frac{T_4}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$

		$\frac{\partial \phi_5}{\partial t} = A_5 \frac{\partial^2 \phi_5}{\partial x^2} + B_5 \phi_4 - C_5 \phi_5$	$\frac{\partial h_5}{\partial t} = \frac{T_5 \cos \varphi}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$	$\frac{\partial h_5}{\partial t} = \frac{T_5}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$
<p>5-AQUIFER SYSTEM</p> <p>• 5 confined aquifers</p>		$\frac{\partial \phi_1}{\partial t} = A_1 \frac{\partial^2 \phi_1}{\partial x^2} - C_1 \phi_1 + E_1 \phi_2$	$\frac{\partial h_1}{\partial t} = \frac{k_1 D_1}{St_1 \cos^2 \varphi} \frac{\partial^2 h_1}{\partial x^2} + \frac{k_{v1}(h_2 - h_1)}{St_1 d_1}$	$\frac{\partial h_1}{\partial t} = \frac{k_1 D_1}{St_1} \frac{\partial^2 h_1}{\partial x^2} + \frac{k_{v1}(h_2 - h_1)}{St_1 d_1}$
		$\frac{\partial \phi_2}{\partial t} = A_2 \frac{\partial^2 \phi_2}{\partial x^2} + B_2 \phi_1 - C_2 \phi_2 + E_2 \phi_3$	$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2}{St_2 \cos^2 \varphi} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2}$	$\frac{\partial h_2}{\partial t} = \frac{k_2 D_2}{St_2} \frac{\partial^2 h_2}{\partial x^2} - \frac{k_{v(1)}(h_2 - h_1)}{d_1 St_2} + \frac{k_{v(2)}(h_3 - h_2)}{St_2 d_2}$
		$\frac{\partial \phi_3}{\partial t} = A_3 \frac{\partial^2 \phi_3}{\partial x^2} + B_3 \phi_2 - C_3 \phi_3 + E_3 \phi_4$	$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3}{St_3 \cos^2 \varphi} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3}$	$\frac{\partial h_3}{\partial t} = \frac{k_3 D_3}{St_3} \frac{\partial^2 h_3}{\partial x^2} - \frac{k_{v(2)}(h_3 - h_2)}{d_2 St_3} + \frac{k_{v(3)}(h_4 - h_3)}{St_3 d_3}$
		$\frac{\partial \phi_4}{\partial t} = A_4 \frac{\partial^2 \phi_4}{\partial x^2} + B_4 \phi_3 - C_4 \phi_4 + E_4 \phi_5$	$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4}{St_4 \cos^2 \varphi} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$	$\frac{\partial h_4}{\partial t} = \frac{k_4 D_4}{St_4} \frac{\partial^2 h_4}{\partial x^2} - \frac{k_{v(3)}(h_4 - h_3)}{d_3 St_4} + \frac{k_{v(3)}(h_5 - h_4)}{St_4 d_4}$
		$\frac{\partial \phi_5}{\partial t} = A_5 \frac{\partial^2 \phi_5}{\partial x^2} + B_5 \phi_4 - C_5 \phi_5$	$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5}{St_5 \cos^2 \varphi} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$	$\frac{\partial h_5}{\partial t} = \frac{k_5 D_5}{St_5} \frac{\partial^2 h_5}{\partial x^2} - \frac{k_{v(4)}(h_5 - h_4)}{d_4 St_5}$

CONCLUSION

In summary, we have formulated a Generalized Hillslope Leakage-dependent, GHL-D model for groundwater flow in n-aquifer system from the Hillslope Leakage-dependent, HL-D groundwater flow models for three-aquifer system. The GHL-D model has been tested on some aquifer types and the results showed that the formulated equation can be applied to modeling groundwater flow in hillslope (sloping) and horizontal single aquifers (unconfined and confined) as well as multi-aquifer systems that contains lower or higher number of aquifers than three. This is the main advantage of this model over the existing groundwater flow models because most of the available models are designed for modeling groundwater flow in either hillslope (sloping) or horizontal aquifers, a single aquifer or multi-aquifer system. This unique feature of GHL-D equation to model groundwater flow in most aquifer types has

made it a more elegant groundwater flow modeling tool with potential for wider applications compared to the existing groundwater flow models. The newly formulated GHL-D Model, when applied to a single aquifer or multi-aquifer system and the resulted groundwater flow equation(s) is/are solved analytically or numerically and used for simulations, will be capable of providing better understanding of the groundwater flow dynamics in such aquifer(s). This is important because most of the sedimentary basins of the world are multi-aquifer in nature and accurate knowledge of groundwater flow within them is important to geoscientists in determining long time yield and water sustainability in such geological structures.

AUTHORS CONTRIBUTIONS

All authors contributed equally to this work.

CONFLICTS OF INTEREST

None

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