



Research Article

On the Properties and Application of Cosine Inverse Exponential Distribution

¹Alhaji Modu Isa, ²Mustapha Mohammed Karagama and ³Alhaji Ismaila Sulaiman
Department of Mathematics and Computer Science, Borno State University, Maiduguri, Nigeria
Department of Mathematics, University of Maiduguri, Borno State, Nigeria
Department of Mathematics, Nasarawa State University, Keffi, Nigeria

*Corresponding author's Email: alhajimoduisa@bosu.edu.ng, doi.org/10.55639/607.222120

ARTICLE INFO:

Keywords:

Cosine-G,
Inverse-Exponential
distribution,
Reliability Analysis,
Maximum-Likelihood
Estimate,

ABSTRACT

Probability distributions are fundamental tools in statistical modeling and data analysis, providing a mathematical framework to describe the behavior of random variables. However, a significant challenge lies in identifying and utilizing a distribution that accurately captures the complexities of diverse real-world data. Many existing distributions may offer robust solutions for specific types of data but fail to generalize across various datasets with different characteristics, such as varying degrees of skewness, kurtosis, and tail behavior. This study was motivated by the attractiveness and flexibility of the Cosine-G family of distributions and the established utility of the Inverse Exponential distribution in various real-life datasets. The research introduces a compound distribution by combining the Cosine-G family of distribution and the Inverse Exponential distribution, resulting in the creation of the Cosine Inverse Exponential distribution. The study explores numerous properties of the newly proposed model, including moments, hazard functions, survival functions, reverse hazard function, cumulative hazard function, and quantile function. The parameters of the model were estimated using the method of maximum likelihood estimate. Exploratory data analysis was conducted on the dataset to examine its characteristics. These datasets exhibited varying degrees of kurtosis and skewness. The proposed model was applied to these datasets and compared with existing distributions. The results demonstrate that the proposed model outperforms its competitors, highlighting its superior flexibility.

Corresponding author: Alhaji Modu Isa, **Email:** alhajimoduisa@bosu.edu.ng
Department of Mathematics and Computer Science, Borno State University, Maiduguri, Nigeria

INTRODUCTION

Statistical distributions are fundamental tools for describing real-world phenomena. Their versatility makes them crucial across various fields, including engineering, medicine, environmental science, and ecology. However, emerging datasets often exhibit characteristics that existing distributions cannot adequately capture. Consequently, many researchers have proposed generalized distributions to address these shortcomings. By compounding classical distributions with generalized ones, sometimes additional parameters are introduced to enhance the ability of some of these classical models to fit real-life data more effectively. The motivation for this study stems from the need to address the limitations of existing probability distributions in capturing the complexities of emerging datasets. While statistical distributions are fundamental tools for describing real-world phenomena across various fields such as engineering, medicine, environmental science, and ecology, many existing models fall short in adequately capturing certain data characteristics. Specifically, the inverse exponential distribution, despite its utility in modeling lifetime data across diverse domains, often lacks the necessary flexibility in its tail and peak behavior. This research aims to overcome these shortcomings by introducing a new probability model that enhances the flexibility and fitting capability of traditional distributions. The primary goal of this study is to develop and introduce a new probability distribution, the Cosine Inverse Exponential distribution, by compounding the Cosine-G family of distributions with the inverse exponential distribution. This new model seeks to provide improved flexibility and better goodness of fit for datasets exhibiting varying degrees of skewness and kurtosis. The study also aims to explore the properties of the proposed distribution, including moments, hazard function, survival function, reverse hazard function, cumulative hazard function, and quantile function, and to compare

its performance with existing models using real-world datasets.

Several notable trigonometric distributions have been introduced in the literature. Some of those recent works include the Sine-Exponential distribution by Isa *et al.*, (2022a), Sine Lomax distribution by Mustapha *et al.*, (2023), Sine Modified Lindley distribution by Tomy and Chesneau (2021), and Sine Burr XII distribution by Isa *et al.*, (2022b). In this research, we focus on the Cosine-G family of distribution developed by Souza *et al.*, (2019) with the inverse exponential distribution developed by Lin *et al.*, (1989). This combination provides enhanced flexibility to traditional distributions due to the inclusion of the trigonometric function in the model.

The classical distribution examined in this study is the inverse exponential distribution, a one-parameter scale distribution introduced by Lin *et al.*, (1989). The inverse exponential distribution is pivotal in modeling and analyzing lifetime data across various domains, including economics and finance, insurance and risk management, quality control, earth sciences, telecommunications, biological sciences, and engineering. Given its significance, numerous extensions of the inverse exponential distribution have been developed. Notable examples include the Transmuted Inverse Exponential distribution by Oguntunde and Adejumo (2015), Inverse Weibull Inverse Exponential distribution by Aldahlan (2019), Extended Inverse-Exponential distribution by Kargbo et al. (2023), Kumaraswamy inverse exponential distribution by Oguntunde *et al.*, (2017), Weibull Inverse Exponential distribution by Chandrakant *et al.*, (2018), Marshall–Olkin alpha power inverse exponential distribution by Basheer (2022), and Extended Generalized Inverse Exponential distribution by (Sule, 2021). Despite these advancements, no single model can perfectly fit all datasets, necessitating further development. The motivation for this research lies in addressing the inverse exponential

distribution's limitations in tail and peak flexibility. While various extensions have been proposed, some lack adequate flexibility. Our goal is to introduce a new extension with the Cosine-G family of distribution, which offers improved flexibility and better goodness of fit for the inverse exponential distribution.

Compounding the Inverse Exponential distribution with the Cosine G family can yield several improvements. This new distribution offers increased flexibility due to additional parameters, enabling it to model a wider range of data shapes. It can provide better tail behavior, offering a more accurate fit for datasets with heavier or lighter tails than those typically modeled by the standard Inverse Exponential distribution. Enhanced control over skewness and

kurtosis makes it suitable for datasets with varying degrees of asymmetry and peakedness. Consequently, these results in a better model fit and improved predictive performance, making the new distribution more versatile for applications in reliability engineering, survival analysis, and other fields requiring the modeling of time-to-event data or lifetimes with complex characteristics. The enhanced flexibility also makes the new distribution more robust to outliers and extreme values, leading to a powerful and versatile tool for statistical modeling.

Inverse Exponential Distribution

The pdf and the cdf of the Inverse Exponential Distribution with parameter θ according to Lin *et al.*, (1989) is defined by:

$$G(x; \theta) = \exp\left(-\frac{1}{\theta x}\right); x > 0, \theta > 0 \quad (1)$$

and the corresponding pdf is given by:

$$g(x; \theta) = \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right), \quad (2)$$

Cosine-G Family of Distribution

The cdf of the Cosine-G family of distribution as defined by Souza *et al.*, (2019) is defined by:

$$F(x; \xi) = 1 - \cos\left\{\frac{\pi}{2} G(x; \xi)\right\} \quad (3)$$

and the corresponding pdf is given by:

$$f(x; \xi) = \frac{\pi}{2} g(x; \xi) \sin\left\{\frac{\pi}{2} G(x; \xi)\right\} \quad (4)$$

where $g(x; \xi)$ and $G(x; \xi)$ are the pdf and cdf of the baseline distribution.

Cosine Inverse Exponential Distribution

The cdf of Cosine Inverse Exponential distribution was obtained by substituting the cdf of the Inverse Exponential in equation (1) into the cdf of the Cosine-G family in equation as follows:

$$F(x; \xi) = 1 - \cos\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\}, x > 0, \theta > 0, \quad (5)$$

and the pdf was obtained by inserting the pdf and cdf in equation (1) and (2) into equation (4) as follows:

$$f(x; \xi) = \frac{\pi}{2} \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right) \sin\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\} \quad (6)$$

The survival function $S(x)$, hazard function $h(x)$, reverse hazard function $r(x)$ and cumulative hazard function $H(x)$ are given in equation (7) to equation (10).

$$S(x) = \cos\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\} \quad (7)$$

$$h(x) = \frac{\frac{\pi}{2} \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right) \sin\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\}}{1 - \cos\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\}} \quad (8)$$

$$r(x) = \frac{\pi}{2} \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right) \tan\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\} \tag{9}$$

$$R(x) = -\ln[S(x)] = -\ln\left\{\cos\left[\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right]\right\} \tag{10}$$

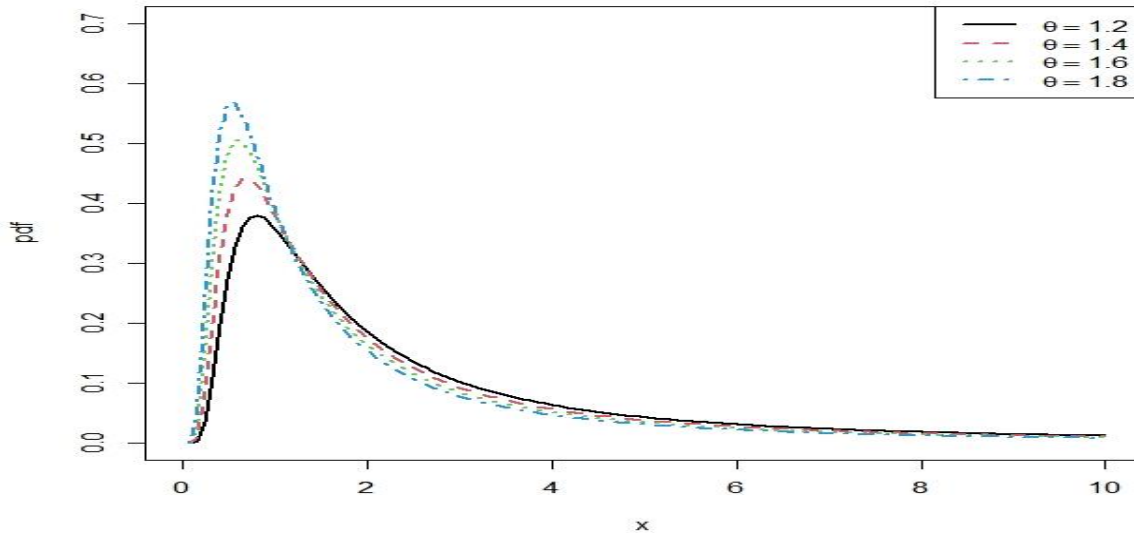


Figure 1: pdf plot of the Cosine Inverse Exponential distribution

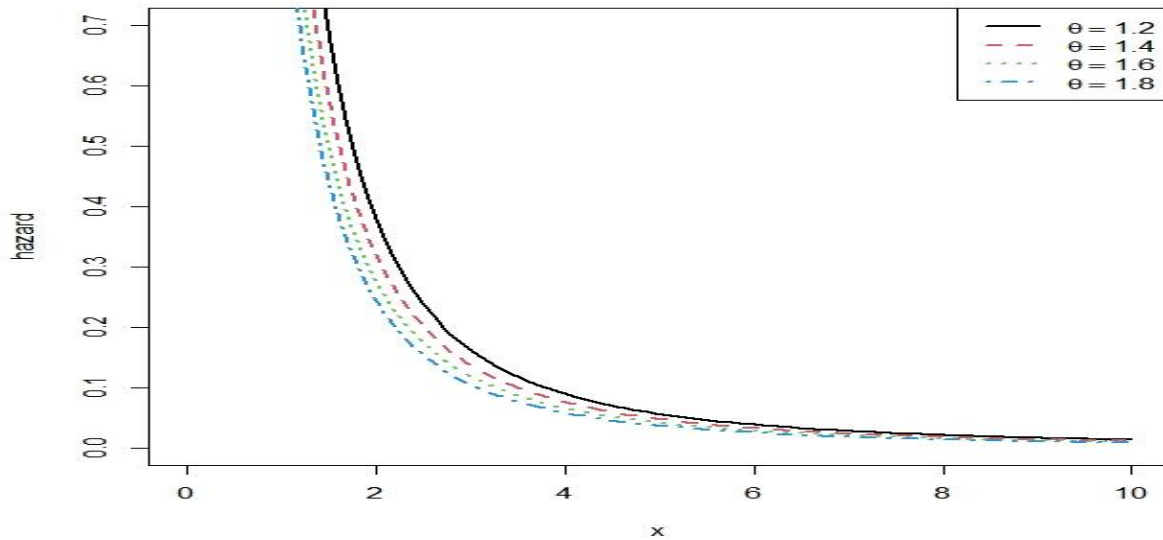


Figure 2: Hazard plot of the Cosine Inverse Exponential distribution

Quantile Function

The quantile function, in statistics, is a mathematical tool that finds the value in a probability distribution below which a specified

proportion of the distribution lies. The quantile function of the Cosine Inverse Exponential distribution is obtained as follows:

$$u = 1 - \cos \left\{ \frac{\pi}{2} \exp \left(-\frac{1}{\theta x} \right) \right\}$$

By isolating the cosine term, the equation becomes and taking the cosine inverse we have:

$$\exp \left(-\frac{1}{\theta x} \right) = \frac{\cos^{-1}(1-u)}{\pi/2}$$

Taking the natural log of both sides, we have:

$$-\frac{1}{\theta x} = \log \left(\frac{\cos^{-1}(1-u)}{\pi/2} \right)$$

By making x the subject of the relations, the quantile function is given by:

$$Q(u) = \left\{ -\frac{1}{\theta \log \left(\frac{\cos^{-1}(1-u)}{\pi/2} \right)} \right\} \tag{11}$$

Density Expansion

The cdf and the pdf of the Cosine Inverse Exponential distribution in equation (5) and (6) can be expanded using Taylor’s series expansion as follows:

$$\sin \left\{ \frac{\pi}{2} \exp \left(-\frac{1}{\theta x} \right) \right\} = \sum_{i=0}^{\infty} \frac{(-1)^i \pi^{2i+1}}{(2i+1)! 2^{2i+1}} \tag{12}$$

Substituting equation (12) into equation (6) we have the following;

$$f(x) = \sum_{i=0}^{\infty} \frac{(-1)^i \pi^{2i+2}}{(2i+1)! 2^{2i+2}} \frac{1}{\theta} x^{-2} \left[\exp \left(-\frac{1}{\theta x} \right) \right]^{2i+1} \tag{13}$$

The pdf in equation (13) can be written as:

$$f(x) = \sum_{i=0}^{\infty} \Psi_i x^{-2} \left[\exp \left(-\frac{1}{\theta x} \right) \right]^{2i+1} \tag{14}$$

where $\Psi_i = \frac{1}{\theta} \frac{(-1)^i \pi^{2i+2}}{(2i+1)! 2^{2i+2}}$

The cdf in equation (5) can also be expanded using the Taylor’s series expansion as follows:

$$\cos \left\{ \frac{\pi}{2} \exp \left(-\frac{1}{\theta x} \right) \right\} = \sum_{j=0}^{\infty} \frac{(-1)^j \pi^{2j}}{(2j)! 2^{2j}} \left[\exp \left(-\frac{1}{\theta x} \right) \right]^{2j} \tag{15}$$

Substituting equation (15) into the cdf in equation (5) gives the following:

$$F(x) = 1 - \sum_{j=0}^{\infty} \frac{(-1)^j \pi^{2j}}{(2j)! 2^{2j}} \left[\exp \left(-\frac{1}{\theta x} \right) \right]^{2j} \tag{16}$$

Therefore, the cdf in equation (16) can be written as:

$$F(x) = 1 - \sum_{j=0}^{\infty} \phi_j \left[\exp\left(-\frac{1}{\theta x}\right) \right]^{2j} \quad (17)$$

$$\text{where } \phi_j = \frac{(-1)^j \pi^{2j}}{(2j)! 2^{2j}}$$

Parameter Estimation

Maximum Likelihood Estimate

Given a sample of n independent observations x_1, x_2, \dots, x_n from Cosine Inverse Exponential distribution, the likelihood function $\ell(\xi; x_1, x_2, \dots, x_n)$ is the product of the pdf evaluated at each observation:

$$\ell(\xi; x_1, x_2, \dots, x_n) = \prod_{i=1}^n f(x_i; \xi) \quad (18)$$

Substituting the given pdf:

$$\ell(\xi; x_1, x_2, \dots, x_n) = \prod_{i=1}^n \left\{ \frac{\pi}{2} \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right) \sin\left[\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right] \right\}$$

The log likelihood function of the pdf in equation (6) is given by:

$$\ell = n \log\left(\frac{\pi}{2}\right) - n \log \theta + \sum_{i=1}^n \log x^{-2} + \sum_{i=1}^n \left(-\frac{1}{\theta x}\right) + \sum_{i=1}^n \log \sin\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\} \quad (19)$$

Differentiating equation (19) partially with respect to θ gives the following:

$$\frac{\partial \ell}{\partial \theta} = -\frac{n}{\theta} + \sum_{i=1}^n \frac{1}{\theta^2 x} + \sum_{i=1}^n \frac{\pi}{2} \frac{1}{\theta x^2} \exp\left(-\frac{1}{\theta x}\right) \cot\left\{\frac{\pi}{2} \exp\left(-\frac{1}{\theta x}\right)\right\} \quad (20)$$

Equation (20) generally requires numerical methods to solve due to its complexity. The solution to equation gives the Maximum Likelihood Estimate of the parameter θ .

Assessing the Consistency of the Parameter Estimate

To assess the performance of the newly proposed Sine Inverse Exponential distribution, a simulation study was conducted using the Monte Carlo Simulation method to compute the mean,

bias and mean root square error of the estimated parameters from the maximum likelihood estimates. The Simulated data is produced using the quantile function defined in equation (11) for different sample sizes: $n = 20, 150, 250, 500$ and replicated 1000 times. For each sample size, $\theta = 0.35, \theta = 0.5, \theta = 1.2,$ and $\theta = 1.5$ were used. Table 1 presents the results of the estimates, bias and root mean square error from the new distribution

Table 1: Estimate, Bias and RMSE of the new Cosine Inverse Exponential Distribution

n	Properties	$\theta = 0.35$	$\theta = 0.5$	$\theta = 1.2$	$\theta = 1.5$	$\theta = 2.0$
20	Est.	0.3518	0.5025	1.2061	1.5076	2.0101
	Bias	0.0018	0.0025	0.0061	0.0076	0.0101
	RMSE	0.0464	0.0663	0.1590	0.1988	0.2650
150	Est.	0.3515	0.5022	1.2052	1.5065	2.0087
	Bias	0.0015	0.0022	0.0052	0.0065	0.0087
	RMSE	0.0732	0.1046	0.2511	0.3139	0.4185
250	Est.	0.3503	0.5004	1.2008	1.5005	2.0007
	Bias	0.0003	0.0004	0.0008	0.0005	0.0007
	RMSE	0.0184	0.0263	0.0794	0.1160	0.1547
500	Est.	0.3502	0.5003	1.2004	1.5010	2.0013
	Bias	0.0002	0.0003	0.0004	0.0010	0.0013
	RMSE	0.0232	0.0331	0.0928	0.0991	0.1324

The table presents results from a Monte Carlo simulation study assessing the performance of the newly proposed Cosine Inverse Exponential distribution. The study computes the mean, bias, and root mean square error (RMSE) for maximum likelihood estimates of parameter θ using different sample sizes (20, 150, 250 and 500) and various true θ values (0.35, 0.5, 1.2, 1.5 and 2.0). The findings indicate that bias and RMSE decrease with increasing sample size, demonstrating improved accuracy and precision in the estimates. For smaller sample sizes, the estimates show higher bias and RMSE, reflecting greater variability. As sample size increases, the estimates converge to the true parameter values, confirming the reliability of the estimator. Thus, the Cosine Inverse Exponential distribution provides consistent and precise parameter estimates, especially with larger sample sizes.

Application

Dataset I

Breaking stress of carbon fibers data of sample 100 observations was considered to test for the efficiency of the model. This data set was used by Isa *et al.*, (2022). The result of the analysis was compared with the baseline distribution (Inverse Exponential) and Exponential distributions for illustrative purposes. We estimated the unknown parameters of each distribution by the maximum-likelihood method. The values of the Akaike information criterion (AIC) and Bayesian

information criterion (BIC) were also computed and compared with the newly developed model. The data are as follows:

3.7, 2.74, 2.73, 2.5, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.2, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.7, 2.03, 1.8, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

Dataset II

The second dataset consists of strengths of 1.5 cm glass fibers, measured at the National Physical Laboratory, England. The data is taken from Smith and Naylor (1987) comprises of 63 observations: 0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.074, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.5, 1.54, 1.6, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.5, 1.55, 1.61, 1.62, 1.66, 1.7, 1.77, 1.84, 0.84, 1.24, 1.3, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.7, 1.78, 1.89.

For each distribution, using maximum likelihood estimates we calculated AIC (Akaike Information Criterion) and BIC (Bayesian Information

Criterion), Corrected Akaike Information Criterion (CAIC), and Hannan-Quinn Information Criterion (HQIC).

Exploratory Data Analysis (EDA)

Exploratory data analysis involves analyzing data to understand the basic structure of datasets. Before applying a new model to a dataset, it is

crucial to investigate the characteristics of the dataset to determine if they align with the features of the newly developed model. Table 1 and 2 presents the descriptive statistics of the two datasets, while Figures 5 and 6 display the box plot, kernel density plot, violin plot, and histogram of the dataset.

Table 2: Summary Statistics of the Breaking Stress of Carbon Fiber Dataset

N	Mean	Skewness	Kurtosis
100	2.6214	0.3682	3.1049

Table 2 presents the summary statistics for a dataset containing the breaking stress of carbon fiber. The skewness value of 0.3682 indicates a slight positive skewness in the dataset. This means that the distribution of the breaking stress values is slightly skewed to the right, with a longer tail on the higher end of the stress values. A positive skewness suggests that there are more

values below the mean and a few relatively high values that pull the mean upwards. The kurtosis value of 3.1049 is very close to 3, which is the kurtosis of a normal distribution (mesokurtic). This suggests that the dataset has a distribution shape similar to a normal distribution in terms of the peak and tail behavior.

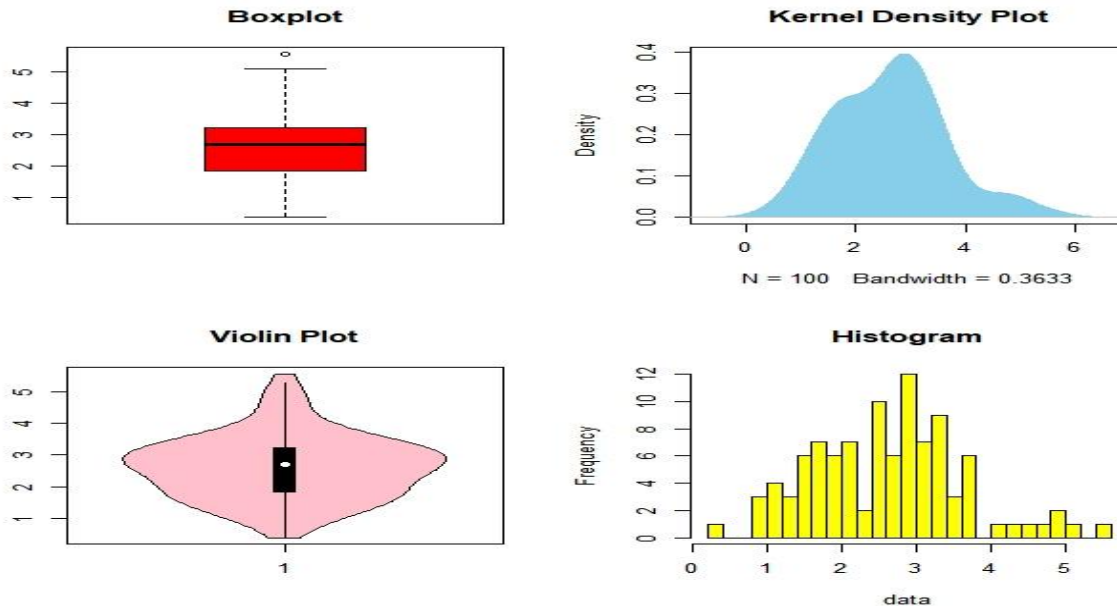


Figure 3: Box-plot, Kernel density, Violin plot and histogram of the first dataset

The analysis of the datasets using various plots reveals a clear positive skewness. Box plots show the interquartile range shifted towards the lower end, indicating data concentration below the median. The kernel density and histogram plots exhibit long right tails, while the violin plot

highlights a wider distribution at the lower end, all confirming positive skewness. This means most data points are clustered towards the lower end with fewer high extreme values. Furthermore, the exploratory data analysis of the breaking stress of carbon fiber dataset also shows

skewness, aligning with the characteristics of the Cosine Inverse Exponential distribution, making

it suitable for modeling this dataset as demonstrated in the pdf plot and Table 3.

Table 3: Summary Statistics of the two datasets

N	Mean	Skewness	Kurtosis
63	1.5068	-0.8999	3.9237

The skewness and kurtosis values provide insights into the characteristics of two datasets. The first dataset, with a skewness of 0.3682, is slightly right-skewed, indicating a longer tail on the right side. In contrast, the second dataset has a skewness of -0.8999, showing moderate left-skewness with a longer tail on the left side. The kurtosis value for the first dataset is 3.1049,

suggesting it is leptokurtic, meaning it has a higher peak and heavier tails than a normal distribution, indicating more extreme values. The second dataset, with a kurtosis of 3.9237, is even more leptokurtic, with a higher peak and heavier tails than the first dataset, indicating an even greater likelihood of extreme values compared to a normal distribution.

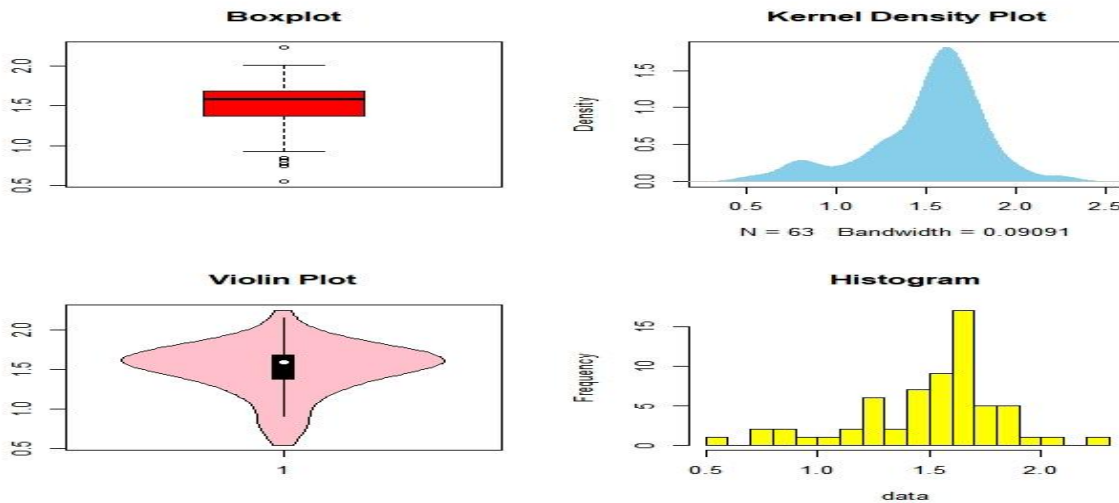


Figure 4: Box-plot, Kernel density, Violin plot and histogram of the second dataset

The four plots in Figures 4 collectively indicate left skewness in the datasets, with a skewness value of -0.8999. The box plots show the interquartile range shifted towards the upper end, and the kernel density plot displays a long left tail, both suggesting a concentration of data above the median. The violin plot also highlights a wider distribution towards the upper end, and the histogram plot shows a longer left tail, reinforcing the evidence of negative skewness.

This suggests most data points cluster towards the upper end with fewer extreme values on the lower end. The exploratory data analysis of the 1.5 cm glass fibers dataset confirms this skewness, which aligns with the proposed Cosine Inverse Exponential distribution shown in the PDF plot in Figure 1, indicating its suitability for modeling the strengths of these glass fibers, as demonstrated in Table 5.

Table 4: MLE, AIC and BIC values of the Breaking Stress of Carbon Fiber Dataset

Model	θ	Loglik(LL)	AIC	BIC	CAIC	HQIC
CIEx	0.8145	-195.01	392.02	394.63	392.06	393.07
Ex	0.4673	199.40	400.79	403.40	400.83	401.85
IEx	0.3815	-196.37	394.74	401.95	394.78	393.08

Table 4 presents the MLE parameter θ , Log-likelihood, AIC, BIC, CAIC and HQIC for the Breaking Stress of Carbon Fiber Dataset using three models: Cosine Inverse Exponential (CIEx), Exponential (Ex), and Inverse Exponential (IEx) distributions. The

CIEx model, with an MLE parameter of 0.8145 has the lowest values for AIC (392.02), BIC (394.63), CAIC (392.06), and HQIC (393.07), indicating it provides the best fit among the three models in fitting breaking stress of carbon fiber dataset.

Table 5: MLE, AIC and BIC values of the of the Strengths of 1.5 cm Glass Fibers Dataset

Model	θ	Loglik(LL)	AIC	BIC	CAIC	HQIC
CIEx	1.2306	-85.775	173.55	175.69	173.62	174.39
Ex	0.7100	-89.439	180.88	183.02	180.02	180.94
IEx	0.6636	-88.830	179.66	185.95	179.73	174.39

Table 5 displays the Maximum Likelihood Estimates, AIC, BIC, CAIC and HQIC values for the Strengths of 1.5 cm Glass Fibers Dataset. The Cosine Inverse Exponential distribution, with an MLE parameter of 1.2306 has the lowest values

for AIC (173.55), BIC (175.69), CAIC (173.62), and HQIC (174.39), indicating it provides the best fit among the three models in fitting Strengths of 1.5 cm Glass Fibers Dataset.

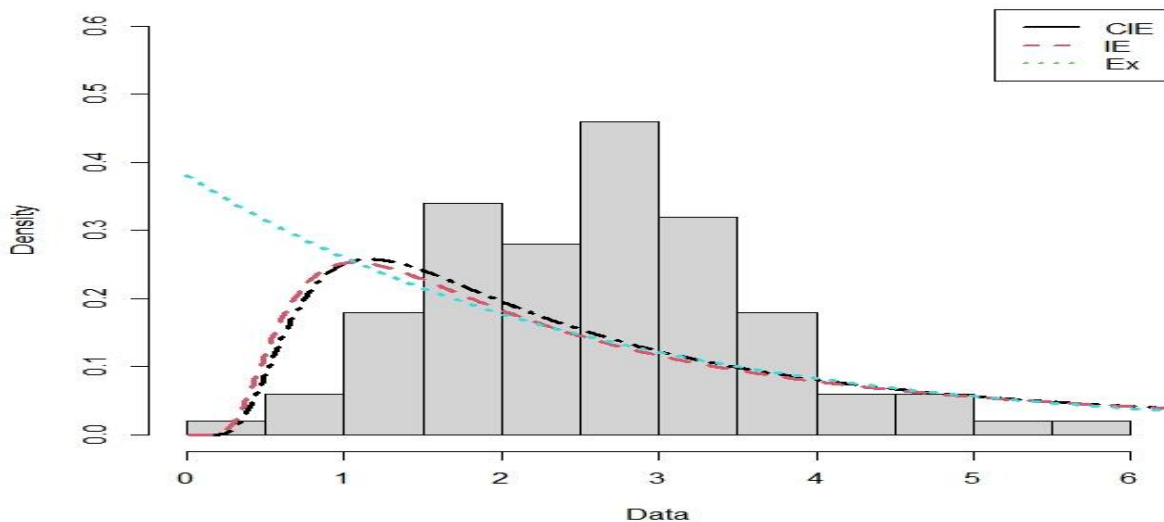


Figure 5: Density plot of Cosine Inverse Exponential distribution for the breaking stress of carbon fiber dataset

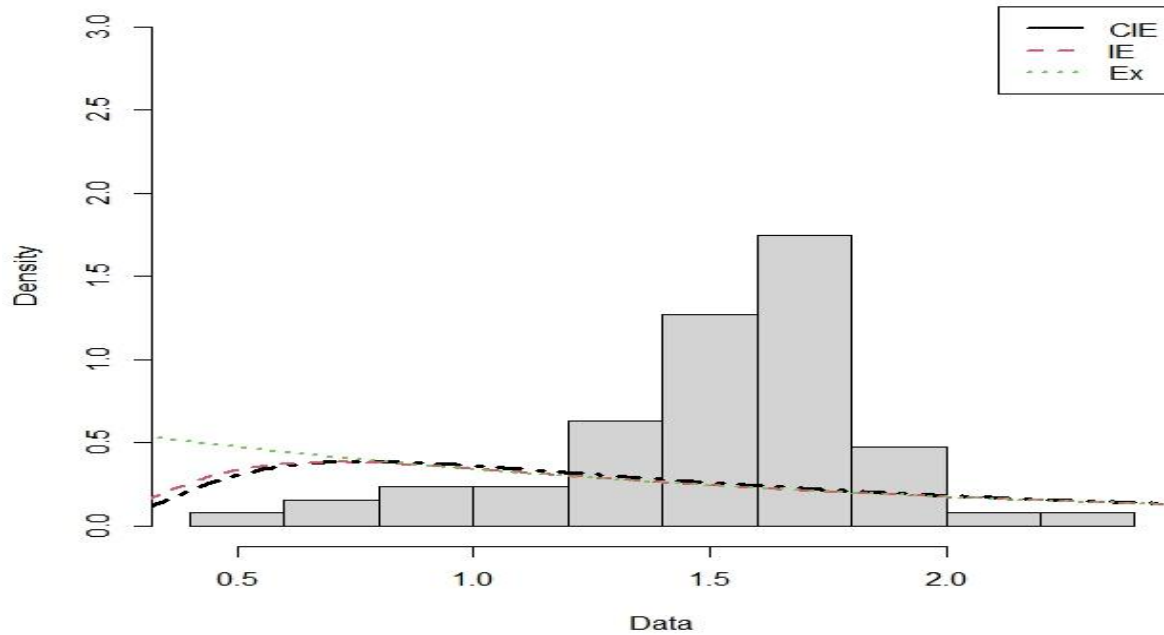


Figure 6: Density plot of Cosine Inverse Exponential distribution for the Strengths of 1.5 cm Glass Fibers Dataset

CONCLUSION

Based on the properties and applications of the Cosine Inverse Exponential distribution, as discussed in this study, we have reached several significant conclusions. Firstly, the Monte Carlo simulation study assessing the performance of the newly proposed distribution reveals promising results. The mean, bias, and root mean square error (RMSE) of the maximum likelihood estimates for different sample sizes and true parameter values demonstrate the distribution's consistency and precision. Notably, as sample size increases, bias and RMSE decrease, indicating improved accuracy and reliability of parameter estimates, especially evident with larger sample sizes. This emphasizes the distribution's robustness and suitability for various datasets, particularly when confronted with differing degrees of skewness and kurtosis. Furthermore, the application of the Cosine

Inverse Exponential distribution to real-world datasets, such as the breaking stress of carbon fibers and strengths of 1.5 cm glass fibers, reinforces its efficacy. Comparative analyses with baseline distributions, namely the Inverse Exponential and Exponential distributions, highlight the superiority of the newly developed model in terms of goodness-of-fit criteria, such as the Akaike and Bayesian information criteria. Additionally, exploratory data analysis confirms the alignment of the proposed distribution with the characteristics of the datasets, further supporting its applicability. In summary, the Cosine Inverse Exponential distribution offers a adaptable and robust framework for modeling diverse datasets, demonstrating superior performance and flexibility compared to existing distributions.

REFERENCES

- Aldahlan, M. A. (2019). The inverse Weibull inverse exponential distribution with application. *International Journal of Contemporary Mathematical Sciences*, 14(1), 17-30.
- Basheer, A. M. (2022). Marshall–Olkin alpha power inverse exponential distribution: properties and applications. *Annals of data science*, 9(2), 301-313.
- Chandrakant, Rastogi, M. K., & Tripathi, Y. M. (2018). On a Weibull-inverse exponential distribution. *Annals of Data Science*, 5, 209-234.
- Isa, A. M., Ali, B. A., & Zannah, U. (2022b). Sine Burr XII Distribution: Properties and Application to Real Data Sets. *Arid. Zone J. Basic Appl. Res*, 1, 48-58.
- Isa, A. M., Bashiru, S. O., Ali, B. A., Adepoju, A. A., & Itopa, I. I. (2022a). Sine-exponential distribution: Its mathematical properties and application to real dataset. *UMYU Scientifica*, 1(1), 127-131.
- Kargbo, M., Gichuhi, A. W., & Wanjoya, A. K. (2023). A novel extended inverse-exponential distribution and its application to COVID-19 data. *Engineering Reports*, e12828.
- Lin, C. T., Duran, B. S., & Lewis, T. O. (1989). Inverted gamma as a life distribution. *Microelectronics and Reliability*, 29(3), 619–626.
- Mustapha, B. A., Isa, A. M., Sule, O. B., & Itopa, I. I. (2023). Sine-Lomax distribution: Properties and Applications to Real Datasets. *FUDMA Journal of Sciences*, 7(4), 60-66.
- Oguntunde, P. E., & Adejumo, A. O. (2015). The transmuted inverse exponential distribution. *International journal of advanced statistics and probability*, 3(1), 1-7.
- Oguntunde, P. E., Adejumo, A. O., & Owoloko, E. A. (2017). Application of Kumaraswamy inverse exponential distribution to real lifetime data. *International Journal of Applied Mathematics and Statistics*, 56(5), 34-47.
- Smith, R. L., & Naylor, J. (1987). A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distribution. *Journal of the Royal Statistical Society Series C: Applied Statistics*, 36(3), 358-369.
- Souza, L., Junior, W. R., Brito, C. C., & Chesneau, C. (2019). General properties for the Cos-G class of distributions with applications. *Eurasian Bulletin of Mathematics*, 2(2), 63–79.
- Sule, B. O. (2021). A new extended generalized inverse exponential distribution: properties and applications. *Asian Journal of Probability and Statistics*, 11(2), 30-46.
- Tomy, L., & Chesneau, C. (2021). The sine modified Lindley distribution. *Mathematical and Computational Applications*, 26(4), 81.